

# Achieving Fast Fractal Image Compression Using Spiral Architecture

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## Abstract

*Spiral Architecture is a relatively new and powerful approach to general-purpose machine vision system. On this novel architecture, Spiral Addition and Multiplication achieve image processing. As we all known, fractal image compression methods have maximal image compression ratio, at the cost of slow coding speed. This paper presents an algorithm to achieve high image compression ratio without slow coding speed on Spiral Architecture, which also improves the Spiral Architecture's usage in image processing.*

Keywords: Image compression, Spiral Architecture, Fractals, Distributed image processing

## 1.0 INTRODUCTION

Fractal image coding is built on the basis of fractal geometry, its feature is the high compression ratio, fast decoding speed, but coding speed is difficult to reach real time processing, in order to solve the problem, at present, the research on combining fractals and other methods is developed.

Spiral Architecture [1] is inspired from anatomical considerations of the primate's vision [2]. From the research about the geometry of the cones on the primate's retina, it is can be concluded that the cones' distribution has inherent organization and is featured by its potential powerful computation abilities. The cones with the shape of hexagons are arranged in a spiral clusters. This clusters consists of the organizational units of vision. Each unit is a set of seven-hexagon [3] as shown in figure1.

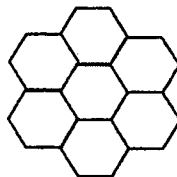


Figure 1. A set of seven-hexagon

In the traditional rectangular image architecture, a set of  $3 \times 3$  rectangles is used as the

unit of vision and each pixel has eight neighbor pixels. In Spiral Architecture any pixel has only six neighbor pixels, which have the same distance to the center hexagon of the seven-hexagon unit of vision. So the spiral architecture has the possibility to save time for global and local processing.

## 2.0 SHM and its operations

A natural data structure that emerges from geometric consideration of the distribution of photoreceptors on the primate's retina has been called the Spiral Honeycomb Mosaic (SHM)[4]. SHM is made up of the hexagonal lattices, which are identified by a designated positive number individually.

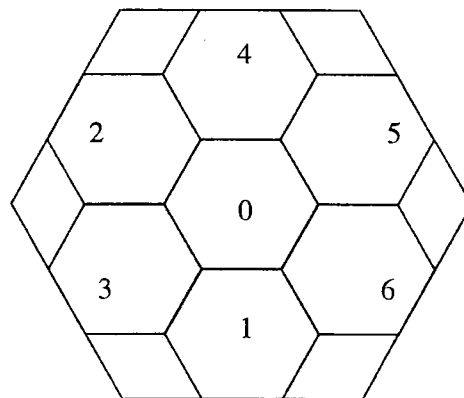


Figure 2. SHM with size of 7

SHM contains very useful geometric and algebraic properties, which can be interpreted in terms of the mathematical object, Euclidean ring. Two algebraic operations have been defined on SHM: Spiral Addition and Spiral Multiplication.

Table 1. Scalar spiral addition

|   | 0 | 1  | 2  | 3  | 4  | 5  | 6  |
|---|---|----|----|----|----|----|----|
| 0 | 0 | 1  | 2  | 3  | 4  | 5  | 6  |
| 1 | 1 | 63 | 15 | 2  | 0  | 6  | 64 |
| 2 | 2 | 15 | 14 | 26 | 3  | 0  | 1  |
| 3 | 3 | 2  | 26 | 25 | 31 | 4  | 0  |
| 4 | 4 | 0  | 3  | 31 | 36 | 42 | 5  |
| 5 | 5 | 6  | 0  | 4  | 42 | 41 | 53 |
| 6 | 6 | 64 | 1  | 0  | 5  | 53 | 52 |

Table 2. Spiral multiplication table

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 0 | 2 | 3 | 4 | 5 | 6 | 1 |
| 3 | 0 | 3 | 4 | 5 | 6 | 1 | 2 |
| 4 | 0 | 4 | 5 | 6 | 1 | 2 | 3 |
| 5 | 0 | 5 | 6 | 1 | 2 | 3 | 4 |
| 6 | 0 | 6 | 1 | 2 | 3 | 4 | 5 |

After image is projected onto SHM, each pixel on the image is associated with a particular hexagon and its SHM address. Multiplication of address  $a$  by the scalar  $\alpha$  ( $\alpha \in \{0,1,\dots,6\}$ ) is obtained by applying scalar multiplication to the components of  $a$  according to the above scalar form, and denoted by,

$$\alpha(a) = (\alpha a_n \alpha a_{n-1} \dots \alpha a_1) \quad \text{Where } a = (a_n a_{n-1} \dots a_1) \text{ for } \forall a_i \in \{0,1,\dots,6\}$$

If the address in Spiral Multiplication is not a scalar,  $\alpha$ , but a common address like,

$$b = (b_n b_{n-1} \dots b_1) \quad \text{For } \forall b_i \in \{0,1,\dots,6\}$$

$$\text{Then } a \times b = \sum_{i=1}^n a \times b_i \times 10^{i-1}$$

where  $\Sigma$  denotes Spiral Addition and  $\times$  denotes Spiral Multiplication. In order to guarantee that all the pixels are still located within the original Spiral area after Spiral Multiplication, a modular multiplication on SHM is defined. Furthermore the transformation through Spiral Multiplication defined on SHM is bijective mapping. That is each pixel in the original image maps one-to-one to each pixel in the output image after Spiral Multiplication.

Modular multiplication is shown as follows, let  $p$  be the product of two elements  $a, b$ .

That is,

$$p = a \times b, a, b \in SHM, \text{ If } p \geq (\text{modulus}), \text{ then}$$

If  $a$  is a multiple of 10 map  $p$  to  $(p + (p \div (\text{modulus}))) \bmod (\text{modulus})$

Otherwise, map  $p$  to  $p \bmod (\text{modulus})$  where  $\text{modulus} = 10^n$

Here, it is assumed that the number of hexagon in Spiral area is  $7^n$ .

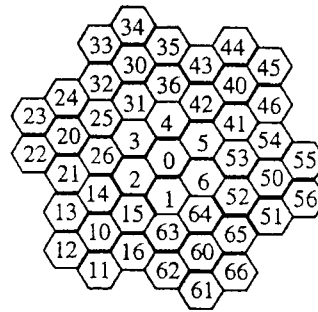


Figure 3. SHM with size of 49

### 3.0 Fast Fractal image coding

With Spiral Addition and Spiral Multiplication, we can easily to separate an original big image into 7 self-similar small images, basically, this method only use the operations to rearrange the pixels, the reason is that within images the Gray level of neighboring pixels is often similar and related. With Spiral Multiplication, we can achieve image rotation without scaling at arbitrary rotation angle [5].

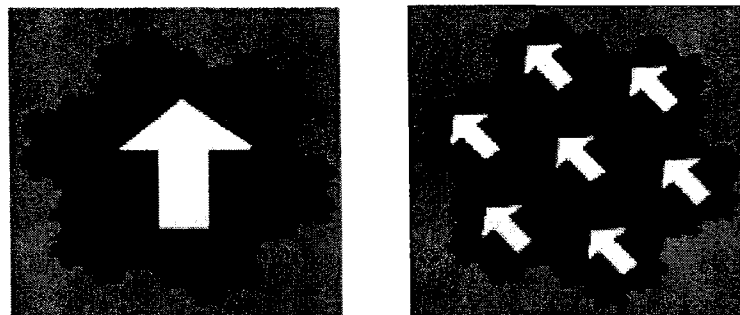


Figure4. One Spiral Multiplication transforms an original image to 7 small similar images

Three steps are developed to realize fractal image compression with fast coding speed on Spiral Architecture.

Step1: Segment Operation

Using Spiral operations to transform the original image into 7 small similar images; using Spiral Multiplication to achieve image rotation without scaling, so that the 7 small images have the same direction as the original one.

Step2: Computation Operation

Computing the average Gray level of the initial image  $\bar{g}$ ; Seeking the average gray

level of the 7small images, noting them as  $g_0, g_1, g_2, g_3, g_4, g_5$  and  $g_6$ ;

**Step3: Coding Operation**

Using the following 7 Affine transformations to code the image

$$\omega_0 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \frac{2}{3} \\ \frac{\sqrt{3}}{3} \\ g_0 - \bar{g} \end{pmatrix}$$

$$\omega_1 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \frac{1}{3} \\ 0 \\ g_1 - \bar{g} \end{pmatrix}$$

$$\omega_2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \frac{1}{6} \\ \frac{\sqrt{3}}{6} \\ g_2 - \bar{g} \end{pmatrix}$$

$$\omega_3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \frac{1}{6} \\ \frac{\sqrt{3}}{2} \\ g_3 - \bar{g} \end{pmatrix}$$

$$\omega_4 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \frac{2}{3} \\ \frac{2\sqrt{3}}{3} \\ g_4 - \bar{g} \end{pmatrix}$$

$$\omega_5 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \frac{7}{6} \\ \frac{\sqrt{3}}{2} \\ g_5 - \bar{g} \end{pmatrix}$$

$$\omega_6 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \frac{7}{6} \\ \frac{\sqrt{3}}{6} \\ g_6 - \bar{g} \end{pmatrix}$$

**4.0 Conclusion**

With this improved technique, the complexity of the encoder is substantially reduced; As we said before, it is a novel method to improve the compressive speed of Fractal image compression method, we did some tests, which show that, with the same PSNR and compression ratio, the new method requires less computation time than the basic methods.(the reference result can be seen in table 3)

Spiral Architecture is really a powerful and practical way to fulfill digital image processing, and the method in our paper is so original that need more time and research to perfect it. We believe more beautiful results related will be proposed in the near future.

Table 3. Comparison of the basic methods and the fast method for "Lena"

|               |            | PSNR  | TIME  |
|---------------|------------|-------|-------|
| Basic methods | Block size |       | 16.86 |
|               | 4 × 4      | 24.08 |       |
|               | 8 × 8      | 28.6  | 22.47 |
|               | 16 × 16    | 35.15 | 31.43 |
| Fast method   |            | 28.08 | 7.23  |

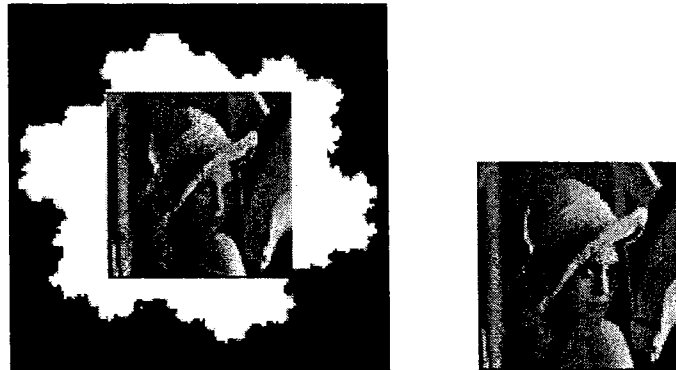


Figure 5. Experimental result

### References:

- [1] P.Sheridan, T.Hintz and W.Moore. "Spiral Architecture in Machine Vision". Proc.Australian Occam and Transputer conference, Amsterdam, 1991.
- [2] E.Schwartz. "Computational anatomy and Functional Architecture of Striate Cortex: spatial Mapping Approach to Perceptual Coding". Vision Research 20: 645-669, 1980.
- [3] P.Sheridan, T.Hintz and D.Alexander. "Pseudo-invariant Image Transformations on a Hexagonal Lattice". Image and Vision Computing, Vol.18:907-917, 2000.
- [4] P.Sheridan, T.Hintz. "Primitive Image Transformations on a Hexagonal Lattice". Charles Sturt University, Bathurst, NSW, Australia, Tech. Report, December 2000.
- [5] Qiang Wu, Xiangjian He and Tom Hintz. "Image Rotation without Scaling on Spiral Architecture". Journal of WSCG, 10(2), Vaclav Skala(ed.):515-520,2002.

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