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# STATIC AND DYNAMIC EVALUATION OF CONTINUITY EFFECT OF CORBELS IN TIMBER BRIDGES

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## ABSTRACT

This paper presents part of the ongoing experimental and analytical work to investigate the effect of corbels on continuity in timber bridges. Full scale beams with corbels were erected for testing the extreme boundary conditions such as simply supported and fixed end conditions. The states of continuity in the test beams were assessed through a three span beam loading test set up. Dynamic tests were performed on the test beams and the modal parameters extracted were compared against those obtained for simply supported beams without corbel. Analytical investigations were onducted and then compared with the experimental results. The static experimental results obtained showed changes in the vertical displacement for different boundary conditions. The magnitude of reduction in the measured vertical displacement was fairly sensitive to the change of boundary conditions. Hence, changes in vertical displacement at mid-span can be used as an indicator for different boundary conditions, which relate to the effect of corbel on both continuity and effective span length in a structural system, especially for timber bridges. A comparative analysis of the first natural frequency for each boundary condition was then undertaken with reference to a simply supported benchmark. The analysis confirmed that the effects of changes in the boundary can be detected using modal parameters.

## 1. INTRODUCTION

Timber bridges in Australia have been in service for hundreds of years. According to Crews *et al.* [1], there are approximately 27,000 timber bridges in Australia, most of which are in excess of 50 years. Some of these bridges are functionally obsolete and structurally deficient. However, many are still in full operation on vital roads and are subjected to continual increment of traffic load. Due aging and defective conditions of these bridges, it is necessary to perform periodical inspections on them to ensure the safety of people using these structures. Many inspection methods are available for the purpose but one novel technique developed by Samali *et al.* [2], namely Dynamic Frequency Analysis (DFA), has proven to be versatile and effective.

However, some aspects of the DFA and even other assessment methods require more attention in order to obtain more reliable results. One of the most important aspects is determining the actual boundary conditions of a bridge as part of the field testing. Morison *et al.* [3] has identified the importance of boundary conditions in obtaining reliable and accurate static and dynamic results. In order to assess the actual condition of any timber bridge, especially a multi-span bridge, a more detailed understanding of its behaviour is necessary. It is therefore essential to carry out investigations on the effects of boundary conditions, in particular the continuity effect of corbel.

## 2. EXPERIMENTAL METHOD

### 2.1 Test Specimen

In this investigation, a full-scale three-span beam was erected using Laminated Veneer Lumber (LVL) with breath and depth of 196mm and 298mm, respectively, and the outer and inner span length of 4,110mm and 4,220mm, respectively. A corbel was placed underneath the beams at inner supports, having the same breath and depth as that of the beams but only measuring 900mm in length. Eight sets of steel rectangular hollow sections (RHS) as well as bolts and nuts were used to simulate different boundary conditions as tabulated in Table 1 and shown in Figure 1. An added top plank made of a piece of LVL, which is 196mm wide, 45mm thick and 900mm long simulates a real timber bridge inner supports configuration.

Table 1: RHS set up for different boundary conditions.

| Boundary conditions | RHSs clamped at positions | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| Simply (S) | √ | x | x | √ |
| Fixed (F) | √ | √ | √ | √ |
| Fixed with an added top plank (FTP) | √ | √ | √ | √ |
| Continuous (C)* | - | - | - | - |

- This case is without corbel and only used in analytical model.

### 2.2 Static Test

The static test set up is illustrated in Figure 1. A five-point loading test was adopted with the loading point set at mid-span of the inner span. Displacement transducers (LVDT) were used to capture the displacements. They were positioned at the soffit of the beam at the locations of quarter, half and three-quarter span on all the spans. The load was generated by a hand-operated hydraulic pump. The displacement and corresponding load were recorded by a data-logger.

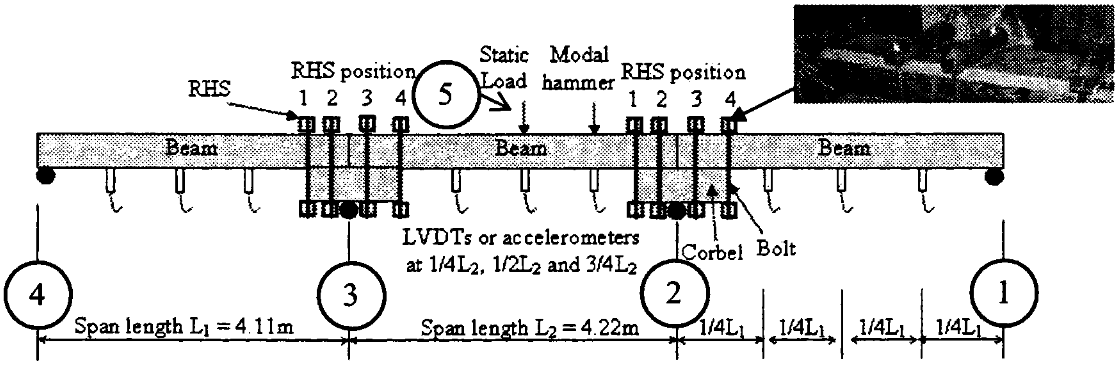


Figure 1: Experimental set up of the three-span continuous beam with corbel.

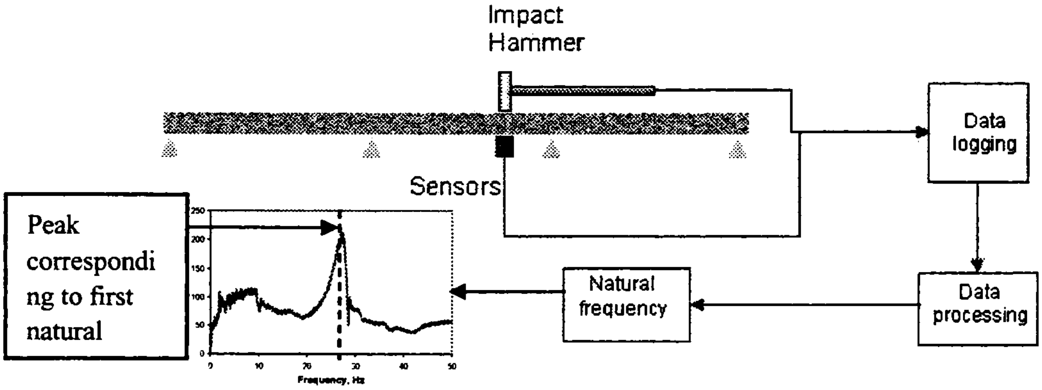


Figure 2: Schematic diagram of the dynamic testing procedure.

### 2.3 Dynamic Test

The beam set up for the dynamic test is similar to that of the static test. It was carried out after the static test. The dynamic testing procedure used is illustrated in Figure 2. It involved the attachment of accelerometers on the soffit of the beams for measuring the acceleration response. The excitation is generated by a modal impact hammer. The resulting dynamic responses are measured with low frequency and high sensitivity accelerometers. The data is logged and is post-processed using an in-house developed software program. The output results are the natural frequencies of the system.

## 3. THE ALGORITHM

### 3.1 Effective Length

Choi *et al.* [4] proposed a correlation between the mid-span deflection δ of the beam to its span length L using the simple beam deflection theory. An assumption is made of the flexural stiffness of the beam, namely, the product of Young's modulus E and the second moment of area I, which have been predetermined. The effective length eventually becomes a function of deflection and span length with the assumption that EI and P are constants as given in Equation 1.

$$L_e = \left( \frac{\delta_a}{\delta_{ss}} L_{ss}^3 \right)^{\frac{1}{3}} \tag{1}$$

where:

$L_e$ is the effective span length for any studied case corresponding to displacement $\delta_a$, and $\delta_{ss}$ and $L_{ss}$ are displacement and span length for an ideal simply supported case, respectively.

### 3.2 Alpha (α) Term

The interaction of corbels and girders in a timber bridge has a significant influence on the structural response of the deck system. In order to take account of end fixity when relating the dynamic stiffness "k" to the static stiffness "EI", Samali *et al.* [5] proposed an α term "equivalency factor", as shown in Equation 2. The α term is a simple method for characterising different boundary conditions as listed in Table 2, in order to convert the stiffness of any given span to an "equivalent" proportion of a simply supported beam. The alpha term equals 1, for the simply supported case. Assuming that k and EI are constant, the α term is eventually a function of the effective length and span length. It is apparent that Equation 3 provides a simple solution for α, which is useful as a boundary condition indicator with respect to the cases studied in this work.

$$k = \frac{48EI}{\alpha L^3} \tag{2}$$

therefore: $\frac{48EI}{\alpha L^3} = \frac{48EI}{L_e^3}$ and: $\frac{1}{\alpha L^3} = \frac{1}{L_e^3}$

$$\alpha = \left( \frac{L_e}{L} \right)^3 \tag{3}$$

Table 2: Variation of theoretical alpha (α) term with "ideal" boundary conditions.

| Boundary condition | α value |
|---|---|
| Pinned end condition | 1.00 |
| Fixed end condition | 0.25 |

### 3.3 First Natural Frequency (FNF)

The First natural frequency (FNF) of a simply supported beam suggested by Clough and Penzien [6] can be expressed as below:

$$f_1 = \frac{\pi}{2L^2}\sqrt{\frac{EI}{\rho A}} \tag{4}$$

where $\rho$ is its density and A the area of cross-section. According to Choi *et al.* [4], with an assumption that EI, $\rho$ and A for a specific case remain unchanged, it is possible to correlate the first natural frequency with span length and effective length based on Equations 3 and 4. It is obvious that the first natural frequency is inversely proportional to the square of span length and, in turn, effective length for a specific boundary condition. The correlation can be extended to estimate an effective length for a measured FNF by comparison to the benchmark case. The relationship is given in Equation 5.

$$L_e^* = \sqrt{\frac{f_1^b}{f_1^*}}L_b \tag{5}$$

where:

$L_e^*$ is the estimated effective length for any studied case corresponding to a FNF of $f_1^*$, while $f_1^b$ and $L_b$ are FNF and effective length of the benchmark case, respectively.

## 4. RESULTS AND DISCUSSIONS

### 4.1 Static Test

From Table 3, the experimental results of mid-span deflection δ at the loading point for boundary conditions of 3-span S, F and FTP are 5.48, 4.85 and 4.15 mm, respectively. It is seen that the rigidity of inner support increases from 3-span S to F, followed by FTP. Using Equation 1, the effective lengths corresponding to the above are 4,178, 4,010 and 3,818 mm, respectively. Referring to Figure 3, these experimental results fall between the two ideal analytical cases of simply-supported 1-span and continuous 3-span boundary conditions. A decreasing trend is observed. This is reasonable as actual boundary conditions are not as flexible as a simply- supported case or as rigid as a continuous case in the ideal analytical simulation.

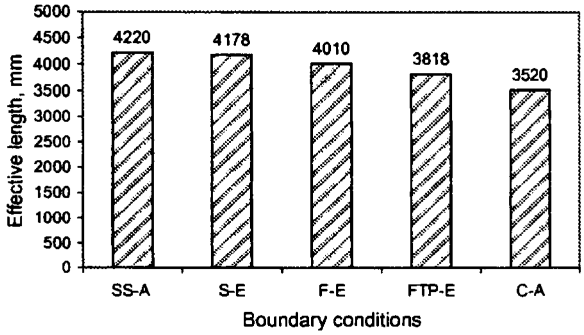
Taking one step further, the effective lengths can be substituted into Equation 3 to obtain the α term as tabulated in Table 3. It is anticipated that the α values for the experimental cases fall between 1 and 0.58 (the ideal cases). This is confirmed by the experimental results of 0.97, 0.86 and 0.74 for 3-span S, F and FTP, respectively. The difference of α value between 3-span S and F and between F and FTP is almost equal, which means the rigidity of continuity has been affected by the RHS and top plank, respectively. It is apparent that corbel inevitably reduces the span length with the analytical 1-span SS as reference.

Table 3: Effective length based on static tests.

| Case | Mid-span deflection, δ | Effective Length, $L_e$ | α |
|---|---|---|---|
| | mm | mm | |
| Analytical | | | |
| 1-span SS* (SS-A) | 5.65 | 4,220 | 1.00 |
| 3-span C** (C-A) | 3.28 | 3,520 | 0.58 |
| Experimental | | | |
| 3-span S (S-E) | 5.48 | 4,178 | 0.97 |
| 3-span F (F-E) | 4.85 | 4,010 | 0.86 |
| 3-span FTP (FTP-E) | 4.18 | 3,818 | 0.74 |

* Standard one-span three-point loading simply supported set up.

** Standard three-span five-point loading simply supported set up.



Note: A and E are analytical solution using a structural analysis software and experimental results, respectively.

Figure 3: Effective length comparison for different boundary conditions.

### 4.2 Dynamic Test

The dynamic test results for the three-span beams are tabulated in Table 4. The First Natural Frequency (FNF) for the 3-span S, F and FTP are 29.0, 33.2 and 34.2 Hertz (Hz), respectively. ased on Equation 4, it is expected that an increase in effective length will reduce the FNF. Alternatively, an increase in the FNF is associated with an increase in supports rigidity.

By employing Equation 5, an attempt to quantify the effects of continuity at the support with corbel was made. The 1-span SS[#] case was chosen as the "benchmark" case to compute the estimated effective length ($L_e$*). From Table 4, the $L_e$* values are ranging from 3,999 to 3,683 mm. These results are reasonably close to the effective length results obtained from static tests with the error of less than 7 percent.

Table 4: The First Natural Frequency (FNF) and estimated effective length.

| Case | Effective length $L_e$ | FNF | Estimated effective length $L_e$* | % different between $L_e$ & $L_e$* |
|---|---|---|---|---|
| | mm | Hz | mm | |
| Experimental | | | | |
| 1-span SS[#] | 4,044 | 26.9 | - | - |
| 3-span S | 4,178 | 29.0 | 3,999 | 4.3 |
| 3-span F | 4,010 | 33.2 | 3,738 | 6.8 |
| 3-span FTP | 3,818 | 34.2 | 3,683 | 3.6 |

[#] Actual length of the test sample is only 4,220mm. The experimental dynamic test was not performed on span length of 4,220mm. Therefore, a simply supported beam with span length of 4,110mm acts as a benchmark case for computation of $L_e$*. The effective length $L_e$ for this benchmark is based on comparison of elastic deflection between analytical and experimental work of 4,110mm span length [4]. As such, it is important to note that a correcting factor of (4,220/4,110) is required as a multiplier by to estimate the estimated effective length $L_e$*.

### 4.3 Analytical Studies

An analytical model was also considered to simulate the effects of corbels in the experimental beam and its contribution to boundary conditions. The modelling was done using a commercially available structural analysis software package. The configuration of the analytical model is depicted in Figure 4. The two inner rigid pins act as a pivot point for corbels to support the beam. The two outer rigid pins simulate the bolting effects between the corbels and the beams.

The deformed shapes of the beam at the loaded span are illustrated in Figure 5. The analytical model, in general, deflects much less than the experimental results indicating that the analytical model is less flexible. This may be due to the analytical model over-estimating support continuity. It is obvious that this simple analytical model is still unable to reflect the experimental realities as the interaction between beam and corbel are complex and hence difficult to analyse.

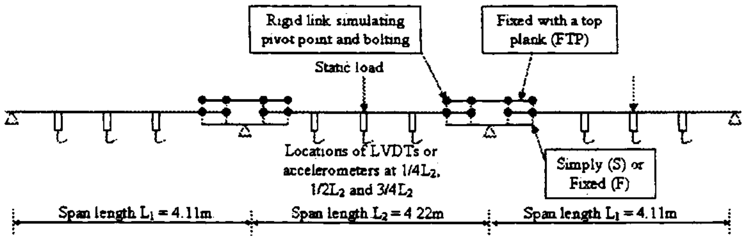
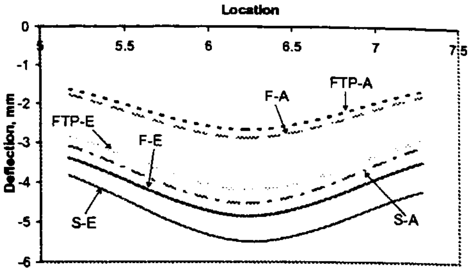


Figure 4. The analytical model.



Note: A and E denote the results obtained from analytical and experimental studies, respectively.
Figure 5. The plots of deflection.

## 5. CONCLUSIONS

The conclusions drawn from the test results presented herein, show that the continuity effects due to presence of corbels cause changes to the span length and to the first natural frequency. The changes in span length have enabled the successful estimation of the effective length and thereafter the $\alpha$ term to represent different boundary conditions. The changes in first natural frequencies are consistent with the degree of rigidity induced at the support and the effective length obtained from static load tests. In addition, the changes in effective lengths and first natural frequencies for all boundary cases investigated can be presented as a boundary condition index. The $\alpha$ term developed from the effective length estimation is also useful to practising engineers to roughly estimate the contribution of boundary effects in a complex structural system such as a real timber bridge. The analytical studies, using a structural analysis model, simulated the continuity effect of corbels. The model by far over-estimated the continuity effect of corbels for the cases of 3-span S, F and FTP. Nevertheless, the model can serve as a useful guide in estimating the continuity effects of corbels.

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