ADAPTIVE SLIDING MODE CONTROL FOR CIVIL STRUCTURES USING MAGNETORHEOLOGICAL DAMPERS

T. H. Nguyen, N. M. Kwok[*], Q. P. Ha, J. Li and B. Samali
Faculty of Engineering
University of Technology, Sydney
Broadway, NSW 2007, Australia
[*]Email: nmkwok@eng.uts.edu.au , Tel: +61-2-9514 7629

Abstract: An adaptive sliding mode controller for vibration control is proposed in this paper for structures embedded with magnetorheological (MR) dampers. Civil structures and buildings are liable to damages during earthquake periods. The application of structural control methodologies is important in order to suppress vibrations due to seismic phenomena and dynamic loading. The use of sliding mode control is accounted for by its robustness to system uncertainties and external disturbances while a MR damper is technologically-efficient for its vibration control and also fail-safe for an ideal semi-active device. The control performance is enhanced by implementing an adaptive control law in estimating the system parameters. Simulation results are included to demonstrate the effectiveness of the proposed controller in a building model under earthquake-like excitations.

Keywords: Structural control; MR damper; adaptive sliding mode control.

1. INTRODUCTION

Earthquake is one of the several disasters which frequently occurs and gives rise to a lot of damages to civil structures. In order to protect these structures including buildings and their occupants, many engineers and researchers have been attracted to the investigation and development for effective approaches in structural control.

A possible strategy to mitigate structural damages is to reduce their vibration magnitudes during an earthquake. With this regard, the use of active mass dampers and semi-active dampers have been proposed and effectively operated in civil structures in more than a decade [1]-[3]. Furthermore, magnetorheological (MR) dampers, as semi-active devices with the advantage of requiring little energy to operate [4], are becoming an attractive candidate in structural control with the incorporation of a suitable controller.

There have been many controller design methods proposed for structural control, such as switching control, pole assignment, linear-quadratic-regulator (LQR) [5, 6]. In recent years, sliding mode control (SMC) has been introduced to this problem domain for its robustness against structural uncertainties, disturbances, actuator non-linearities and hysteresis [7]-[10]. Recently, a quantised-sliding mode controller, using MR dampers, was proposed for structural control [11]. The performance was improved but not preserved well over a large range of parametric variations.

In this work, the robust control problem for civil structures under earthquake excitations is addressed by using the adaptive SMC methodology [12]-[15]. The SMC approach guarantees system robustness while the adaptive control law will enhance the system insensitivity to parametrised non-linearity and hysteresis arising from MR dampers.

The paper is organized as follows. In Section 2, the physical characteristics of an MR damper are briefly described. In Section 3, the control system of a building structure is modelled using the motion equation consisting of non-linear inputs and disturbances. In Section 4 and 5, the control design and an adaptive algorithm are developed with the objective to generate the required damping force for the structure. In Section 6, numerical simulation for the system with MR dampers are presented to illustrate the effectiveness of the proposed control technique. Finally, Section 7 concludes the paper.

2. MAGNETORHEOLOGICAL DAMPER

An MR damper contains nanoscale magnetizable particles suspended in a carrier ferro-fluid. Under the application of a magnetic field the particles are aligned in chain-like structures [16, 17], thus, producing controllable damping forces (see Fig. 1). There are many types of MR damper models available such as the Bingham viscous-plastic model [18-20], the Bouc-Wen model [21], the modified Bouc-Wen model [4], and recently a model that is described by explicit expressions [22]. The latter is adopted here to result in simpler system dynamics suitable for the control design.

The equations of the MR damper are presented as

$$f = c\dot{x} + kx + \alpha z + f_0, \tag{1a}$$

$$z = \tanh(\beta\dot{x} + \delta sign(x)), \tag{1b}$$

where x is the damper diaphragm displacement, f is the output force, z is the hysteresis function, f_0 is the damper force offset, $\beta = 0.09$ is a constant against the supplied current values, α is the scaling parameter and c, k are the viscous and stiffness coefficients. Parameters are described as functions of the supply current, i, as [22]:
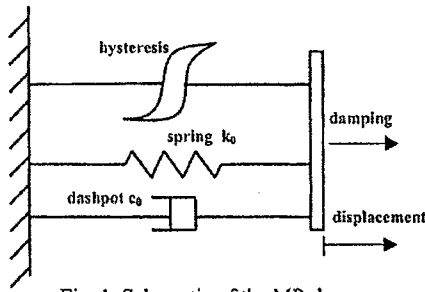
Fig. 1. Schematic of the MR damper

$$c = c_1 i + c_0 = 3.32i + 0.78, \tag{1c}$$

$$k = k_1 i + k_0 = -i + 3.97, \tag{1d}$$

$$\alpha = \alpha_2 i^2 + \alpha_1 i + \alpha_0 = -264 i^2 + 939.73i + 45.86, \tag{1e}$$

$$\delta = \delta_1 i + \delta_0 = 0.44i + 0.48, \tag{1f}$$

$$f_0 = h_1 i + h_0 = -18.21i - 256.50. \tag{1g}$$

3. CONTROL OF CIVIL STRUCTURES

Consider the system of an n-storey building structure subjected to earthquake excitation $\ddot{x}_g(t)$ as shown in Fig. 2. The proposed control system, installed at the structure, consists of MR dampers, controller and current driver. When structural vibration is induced by earthquake $\ddot{x}_g(t)$, the controller with the current driver will excite the MR dampers and the forces f will be generated to eliminate the vibration of the structure. The responses to be regulated are the displacements, velocities, and accelerations ($x, \dot{x}, \ddot{x}$) of the structure, where x is the displacement of the floors.
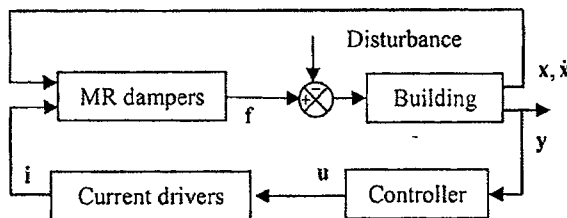

Fig. 2. Block diagram of the control system

The equation of motion of the structure is

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = \Gamma f(t) + M\Lambda \ddot{x}_g(t), \tag{2}$$

where $x(t) = [x_1\ x_2, ..., x_n]^T$, $x(t) \in R^n$ is an n-vector of the displacements, $f(t) \in R^r$ is a vector consisting of the control forces, $\ddot{x}_g(t)$ is the earthquake excitation acceleration, and matrices $M \in R^{n \times n}$, $C \in R^{n \times n}$, $K \in R^{n \times n}$ are respectively the mass, damping and stiffness. Matrix $\Gamma \in R^{n \times r}$ denotes the location of r dampers, and $\Lambda \in R^n$ is a vector indicating the directional influence of the earthquake excitation.

Equation (2) can be rewritten in the state-space form as

$$\dot{z}(t) = Az(t) + B_0 f(t) + E_0(t), \tag{3}$$

where $z(t) \in R^{2n}$ is the state vector, $A \in R^{2n \times 2n}$ is the system matrix, $B_0 \in R^{2n \times r}$ is a constant gain matrix and $E_0(t) \in R^{2n}$ is a disturbance vector, respectively. They are given by

$$z(t) = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \tag{4a}$$

$$B_0 = \begin{bmatrix} 0 \\ M^{-1}\Gamma \end{bmatrix}, \quad E_0(t) = \begin{bmatrix} 0 \\ \Lambda \end{bmatrix} \ddot{x}_g(t). \tag{4b}$$

From Eqs. (1a)-(1g) and with one MR damper, we can rewrite the force equation of the MR damper as

$$
\begin{aligned}
f &= (c_1\dot{x} + k_1 x + h_1 + \alpha_1 z)i + (c_0\dot{x} + k_0 x + h_0 + \alpha_0 z + \alpha_2 i^2 z) \\
&= B_1 i + D_1,
\end{aligned}
\tag{5}
$$

where $B_1 = c_1\dot{x} + k_1 x + h_1 + \alpha_1 z$, and

$$D_1 = c_0\dot{x} + k_0 x + h_0 + \alpha_0 z + \alpha_2 i^2 z$$

are non-linear functions.

Assume that the MR dampers are installed at r floors of the structure, the equation of the MR damper at floor j ($j=1,2,...,r$) can be rewritten as follows

$$f_j = B_j(x_j)i_j + D_j(x_j, z_j, i_j), \tag{6}$$

or

$$f = B^* i + D^*, \tag{7}$$

where $f \in R^r$ is the vector of damping forces, $i \in R^r$ is the vector of control currents, $D^* \in R^r$ and $B^* \in R^{r \times r}$ is a diagonal matrix.

Substitution of Eq. (7) into Eq. (3) leads to the following

$$\dot{z} = Az + B_0(B^* i + D^*) + E_0. \tag{8}$$

The state space equation can be written as

$$\dot{z} = Az + Bi + E, \tag{9}$$

where $B_0 B^* = B + \Delta B$, $B \in R^{2n \times r}$ is a known gain matrix, $E = B_0 D^* + E_0 + \Delta Bi$, and $E \in R^{2n}$ is the vector of unknown disturbance.

4. SLIDING MODE CONTROL

The main advantage of the SMC is its robustness against variations in system parameters or external disturbance. The selection of the control gain is related to the magnitude of uncertainty in order to keep the trajectory on the sliding surface. For simplicity, let $\sigma \in R^r$ be an r-dimensional sliding function consisting of a linear combination of the state variables, i.e.

$$\sigma = Sz, \tag{10}$$

where $S \in R^{r \times 2n}$ is a matrix to be determined such that the motion on the sliding surface $\sigma = 0$ is stable.

The controller output i consists of two components

$$i = i_e + i_s,$$ (11)

where i_e and i_s are respectively the equivalent control and the switching control.

Assuming the availability of the state vector $z(t)$, and the controllability of the system (A,B), by defining a cost function

$$J = \int z^T Q z \, dt,$$ (12)

then upon the choice of a positive definite matrix Q, one can obtain the LQR gain F [6], from which the sliding matrix S can be derived to result in the equivalent control

$$i_e = -Fz.$$ (13)

Indeed, by neglecting the disturbance E, and substitution of $i = i_e$ into the time derivative of the sliding function, one has

$$\dot\sigma = S\dot z = S(Az + Bi_e),$$ (14)

from which condition $\dot\sigma = 0$ yields

$$i_e = -(SB)^{-1}SAz.$$ (15)

Thus matrix S can be chosen [9] such that the equivalent dynamics in the sliding mode will satisfy the optimal criterion (12). Now, in order to design the switching control, let us first assume the following matching conditions:

$$E = B\varepsilon \quad \text{and} \quad \|\varepsilon\| \le \rho_E,$$ (16)

where $\rho_E > 0$ is a known positive value.

Consider the Lyapunov function $V_a = 0.5\sigma^T\sigma$. Substituting eqs. (9) and (11) into its time derivative gives

$$\dot V_a = \sigma^T S(Az + Bi + E)$$
$$= \sigma^T S(Az + B(i_e + i_s) + E)$$
$$= \sigma^T S(Az + Bi_e) + \sigma^T S(Bi_s + E),$$

where $\sigma^T S(Az + Bi_e) = 0$ as of (15). Thus, from (16) one has

$$\dot V_a = \sigma^T SB(i_s + \varepsilon) \le \sigma^T SBi_s + \|\sigma^T SB\|\rho_E.$$ (17)

To satisfy the sliding condition $\dot V_a = \sigma^T \dot\sigma < 0$, the switching control is proposed as

$$i_s = -\rho\frac{B^T S^T \sigma}{\|\sigma^T BS\|},$$ (18)

where $\rho > \rho_E$ is a known control gain. The selection of this control gain depends on the expected uncertainty in the unknown disturbance, with a trade-off between the robustness property of the controller and the chattering effect due to the discontinuity of the switching law (18).

5. ADAPTIVE SLIDING MODE CONTROL

In the control of quake-induced structures, as parametric uncertainties and external disturbances are very difficult to expect or estimate, a large value of the control gain ρ is most likely selected, resulting in an unnecessary deviation from the sliding surface, a large magnitude of discontinuity in the control gain and hence associated difficulties in implementation of the controller. Therefore, an appropriate mechanism to adjust the control gain can help addressing this problem. Among available techniques for tuning the discontinuous control signal, an on-line adjustment of the control gain with a self-tuning adaptive law seems to be suitable. For this, Fig. 3 shows the system block diagram illustrating this idea.
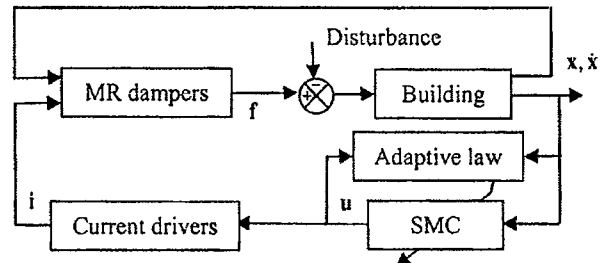

Fig. 3. Block diagram with adaptive sliding mode

The SMC law (11) will be used, where the equivalent control is given by (15) and the switching control (18) is now using the estimate $\hat\rho(t)$ of the control gain ρ, i.e.

$$i_s = -\hat\rho\frac{B^T S^T \sigma}{\|\sigma^T BS\|},$$ (19)

where $\hat\rho(t)$ is subject to the adaptive law

$$\dot{\hat\rho}(t) = \gamma^{-1}\|\sigma^T BS\|,$$ (20)

and $\gamma > 0$ is the adaptation gain.

By introducing the gain error

$$\tilde\rho(t) = \hat\rho(t) - \rho,$$ (21)

and using the Lyapunov function

$$V = 0.5(\sigma^T\sigma + \gamma\tilde\rho^2),$$ (22)

with the notice that $\dot{\tilde\rho} = \dot{\hat\rho}$, one can obtain

$$\dot V = (\sigma^T\dot\sigma + \gamma\tilde\rho\dot{\hat\rho})$$
$$= \sigma^T SB(i_s + \varepsilon) + (\hat\rho - \rho)\|\sigma^T BS\|.$$

Now substitution of the adaptive switching control law (19) in the above equation for the time derivative of the Lyapunov function gives

$$\dot{V} = -\hat{\rho}\frac{\sigma^T SBB^T S^T \sigma}{\|\sigma^T BS\|} + \sigma^T SB\varepsilon + (\hat{\rho} - \rho)\|\sigma^T BS\|$$

$$= \sigma^T SB\varepsilon - \rho\|\sigma^T BS\| \le -\|\sigma^T BS\|(\rho - \|\varepsilon\|), \qquad (23)$$

which results in $\dot{V} < 0$ according to assumption (16) and the selection $\rho > \rho_E$.

From (23), as $\dot{V}(z,t) = \sigma^T SB\varepsilon - \rho\|\sigma^T BS\|$ is negative semi-definite and $V(z,t)$ is lower bounded, if $\dot{V}(z,t)$ is uniformly continuous in time, as is the case for structural systems with MR dampers using model (9), then according to Barbalat's lemma, $\dot{V}(z,t) \to 0$ as $t \to \infty$. Thus,

$$\lim_{t\to\infty} z^T S^T SB(\varepsilon - \rho B^T S^T \sigma/\|\sigma^T BS\|) = 0, \quad \text{and} \quad \text{hence,}$$

$z \to 0$ as $t \to \infty$. Note that for a convergence of the estimated value $\hat{\rho}(t)$ to the upper bound ρ, the so-called persistent excitation condition [12] should be satisfied.

6. SIMULATION RESULTS

Consider the structure of a five-storey building model which has two MR dampers installed at the first floor and the second floor as shown in Fig. 4, $x = [x_1, x_2, x_3, x_4, x_5]^T$ is the displacement vector, f_1 and f_2 are forces of these MR damper and parameters m_i, k_i, c_i $(i = 1,2,...,5)$ are mass, damping and stiffness coefficients, respectively.
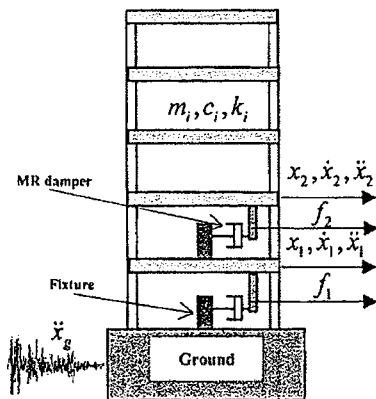

Fig. 4. The building model with 2 MR dampers

The corresponding matrices $M, C,$ and K are as follows

$$M = \begin{bmatrix} 337 & 0 & 0 & 0 & 0 \\ 0 & 330 & 0 & 0 & 0 \\ 0 & 0 & 330 & 0 & 0 \\ 0 & 0 & 0 & 330 & 0 \\ 0 & 0 & 0 & 0 & 337 \end{bmatrix} kg,$$

$$C = \begin{bmatrix} 225 & -157 & 26 & -7 & 2 \\ -157 & 300 & -126 & 25 & -4 \\ 26 & -126 & 299 & -156 & 16 \\ -7 & 25 & -156 & 279 & -125 \\ 2 & -4 & 16 & -125 & 125 \end{bmatrix} \frac{Ns}{m},$$

$$K = \begin{bmatrix} 3766 & -2869 & 467 & -234 & 27 \\ -2869 & 5149 & -2959 & 446 & -70 \\ 467 & -2959 & 5233 & -2836 & 280 \\ -234 & 446 & -2836 & 4763 & -2277 \\ 27 & -70 & 280 & -2277 & 2052 \end{bmatrix} \frac{kN}{m}.$$

Accelerations $\ddot{x}_1$ and $\ddot{x}_2$ corresponding to these MR dampers installed at the building are

$$\ddot{x}_1 = A_{01} + B_1 f_1 + E_{01}$$
$$= A_{01} + B_1(h_1 i_1 + D_1) + E_{01}$$
$$= A_{01} + B_1 h_1 i_1 + B_1 D_1 + E_{01}$$
$$= A_{01} + B_1 h_1 i_1 + E_1,$$

$$\ddot{x}_2 = A_{02} + B_2 f_2 + E_{02}$$
$$= A_{02} + B_2(h_2 i_2 + D_2) + E_{02}$$
$$= A_{02} + B_2 h_2 i_2 + B_2 D_2 + E_{02}$$
$$= A_{02} + B_2 h_2 i_2 + E_2,$$

where

$$A_{01} = -m_1^{-1}((k_1 + k_2)x_1 - k_2 x_2 + (c_1 + c_2)\dot{x}_1 - c_2 \dot{x}_2),$$

$$A_{02} = -m_1^{-1}((k_1 + k_2)x_1 - k_2 x_2 + (c_1 + c_2)\dot{x}_1 - c_2 \dot{x}_2),$$

$$E_1 = B_1 D_1 + E_{01}, \quad E_2 = B_2 D_2 + E_{02},$$

E_1 and E_2 are respectively first-floor and second-floor disturbances, while B_1 and B_2 are current gains. Control parameters are given in Table 1.

Table 1: Control parameters

Damper No.	SMC	Adaptive SMC	
	ρ	ρ	γ
1	650	500	1
2	1.5	0.45	[0.1; -0.5]

Time responses of floor displacements are shown in the Figs. 5-12. Shown in Fig. 5 is the El-Centro earthquake record. Fig. 6 is for the floor displacement without control. Fig. 7 and Fig. 8 are the floor displacements using SMC and adaptive sliding mode control (ASMC) for one MR damper installed at the 1st floor, respectively. Fig. 9 and Fig. 10 are floor displacements using SMC and ASMC for two MR dampers installed at the 1st and 2nd floor. Fig. 11 and Fig.12 are control forces f_1, f_2 of the MR dampers installed at floor-1 and floor-2. In addition, Table 2 summarises numerical results of cases such as uncontrolled, SMC-1 MR damper, ASMC-1 MR damper, SMC-2 MR dampers, ASMC-2 MR damper. It is observed that using ASMC with 2 MR dampers installed at the 1st and 2nd floor give the most satisfactory results.
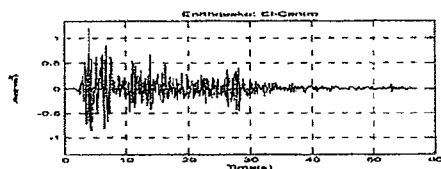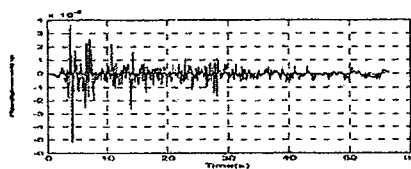
Fig. 5. Earthquake record: El-Centro

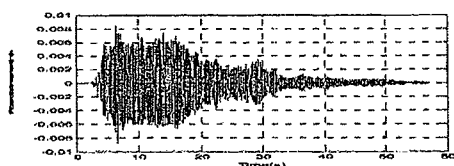Fig. 9. Floor displacement using SMC-2 MR dampers

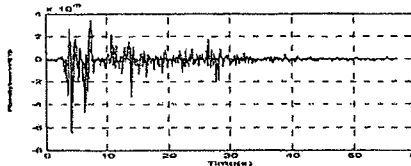Fig. 6. Floor displacement-uncontrolled

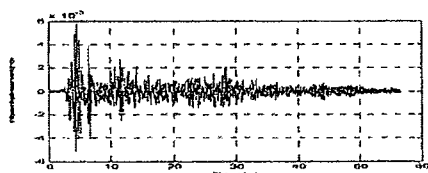Fig. 10. Floor displacement using ASMC-2 MR dampers

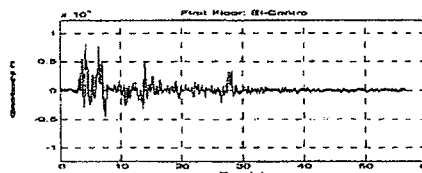Fig. 7. Floor displacement using SMC-1 MR damper

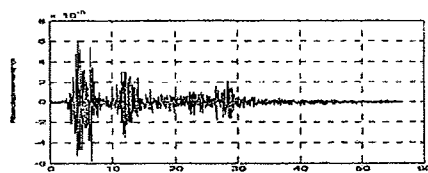Fig. 11. Control force of MR damper at first-floor
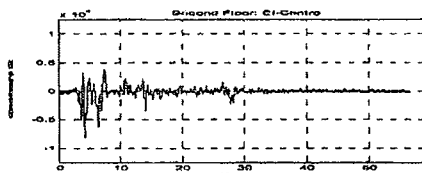

Fig. 8. Floor displacement using ASMC-1 MR damper

Fig. 12. Control force of MR damper at second-floor

Table 2. Floor displacements from different controls

Floor No.	Uncontrolled		SMC-1 damper		ASMC-1 damper		SMC-2 dampers		ASMC-2 dampers	
	Max (mm)	RMS (mm)	Max (mm)	RMS (mm)	Max (mm)	RMS (mm)	Max (mm)	RMS (mm)	Max (mm)	RMS (mm)
1	6.3	2.2	3.0	0.20	3.3	0.23	2.8	0.40	2.5	0.35
2	9.0	1.7	3.8	0.41	3.4	0.28	3.0	0.50	3.1	0.45
3	12.0	2.0	4.0	0.38	3.2	0.27	3.3	0.60	3.4	0.50
4	13.5	2.2	5.1	0.39	5.0	0.30	4.3	0.60	4.2	0.50
5	13.5	2.3	5.1	0.37	5.0	0.32	4.2	0.52	4.1	0.47

7. CONCLUSION

An adaptive sliding mode controller has been proposed to for semi-active control of civil engineering structures, embedded with magnetorheological dampers, under a dynamic loading source such as earthquake excitations. While SMC is robust to system uncertainties and disturbances, the performance of the system under control is enhanced by adaptively estimating the control gain. The system stability is proved on the basis of the Lyapunov stability theory. Extensive simulation results from applying the proposed controller to a five-storey model building have illustrated the effectiveness of the proposed method.

REFERENCES

[1] C. C. Chang, Henry T. Y. Yang, "Control of building using active tuned mass dampers," *Journal of Engineering Mechnics*, Vol. 121, No. 3, pp. 355-366, 1995.

[2] B. F. Spencer Jr, S. J. Dyke, H. S. Deoskar, "Benchmark problem in structural control part I: Active mass drive system," *Earthquake Engineering and Structural Dynamics*, Vol. 27, pp. 1127-1139, 1998.

[3] M. D. Symans, M. C. Constantinou, "Semi-active control for seismic protection of structures: a state-of-the-art review," *Engineering Structures*, Vol. 21, pp. 469-487, 1999.

[4] B. F. Spencer Jr., S. J. Dyke, M. K. Sian and J. D. Carlson, "Phenomenological model for magnetorheological dampers," *Journal of Engineering Mechanics*, pp. 230-238, 1997.

[5] J. N. Yang, J. C. Wu, A. K. Agrawal and Z. Li, "Sliding mode control for seismic-excited linear and nonlinear civil engineering structures," Technical Report NCEER-94-0017, *University of California*, Dept. of Civil Engineering, Irvine, California, pp. 1-11, 1994.

[6] Q. P. Ha, J. Li, G. Hong and B. Samali, "Active structural control using dynamic output feedback sliding mode," *Proc. 2001 Australian Conf. on Robotics and Automation*, Sydney, Nov 2001, pp. 20-25, 2001.

[7] R. A. DeCarlo, S. H. Zak and R. A. Matthews, "Variable structure control of nonlinear multivariable system: A tutorial," *IEEE Proceedings*, Vol. 76, No. 3, pp. 212-232, 1988.

[8] V. Utkin, J. Guldner, and J. Shi, *Sliding Mode Control in electromechanical systems*, Taylor and Francis, UK, 1999.

[9] C. Edwards and S. K. Spurgeon, *Sliding mode control: theory and applications*, Taylor and Francis, London, UK, 1998.

[10] R. Adhikari and H. Yamaguchi, "Sliding mode control of building with ATMD," Earthquake Engineering and Structural Dynamics, Vol. 26, pp. 409-422, 1997.

[11] N. M. Kwok, T. H. Nguyen, Q. P. Ha, J. Li and B. Samali, "MR damper structure control using a multi-level sliding mode controller," *Australian Earthquake Engineering Society Conference* (AEES), Albury Australia, pp. 10.1-10.7, 2005.

[12] J.J.E. Slotine and W. Li, *Applied nonlinear control*, Prentice Hall, Englewood Cliffs, New Jersey, 1991.

[13] F. Mark, S. Csaba, and R. Eric, *Performance of nonlinear approximate adaptive controllers*, John Wiley and Sons, Hoboken, New Jersey, 2003.

[14] R. J. Wai, J. D. Lee, and L. J. Chang, "Development of adaptive sliding mode control for nonlinear dual-axis inverted-pendulum system," *EEE/ASME International Conference on Advanced Intelligent Mechatronics*, Vol. 2, pp. 815-820, 2003.

[15] R. J. Wai, "Adaptive sliding-mode control for induction servomotor drive," *IEE Electric Power Applications*, Vol. 147, Issue 6, Pg. 553-562, 2000.

[16] G. Bossis and S. Lacis, "Magnetorheological fluids," *Journal of Magnetism and Magnetic Materials*, Vol. 252, pp. 224-228, 2002.

[17] S. L. Djajakesukuma, B. Samali and H. Nguyen, "Study of a semi-active stiffness damper under various earthquake inputs," *Earthquake Engineering Structural Dynamics*, Vol. 31, pp. 1757-1776, 2002.

[18] T. Butz and O. von. Stryk, "Modelling and simulation of electro- and magnetorheological fluid dampers," *Applied Mathematics and Mechanics*, pp. 3-20, 2002.

[19] R. Stanway, J. L. Sproston and N. G. Stevens, "Non-linear modelling of an electrorheological vibration damper," *Journal of Electrostatics*, Vol. 20, pp. 167-184, 1987.

[20] Y. K. Wen, "Method of random vibration of hysteretic systems," *Journal of Engineering Mechanics Division*, ASCE, Vol. 102, pp. 249-263, 1976.

[21] S. J. Dyke, B. F. Spencer Jr, M. K. Sain and J. D. Carlson, "Modelling and control of magnetorheological dampers for seismic response reduction," *Smart Material Structure*, Vol 5, pp. 565-575, 1996.

[22] N. M. Kwok, Q. P. Ha, T. H. Nguyen, J. Li and B. Samali, "A novel hysteretic model for magnetorheological fluid dampers and parameter identification using particle swarm optimisation," *Sensors and Actuators A*, (in press), 2006.