# **Active Structural Control Using Dynamic Output Feedback Sliding Mode**

# Q. P. Ha, Jianchun Li, Guang Hong, and Bijan Samali

Faculty of Engineering, The University of Technology, Sydney PO Box 123, Broadway 2007, Australia.

{quang.ha, jianchun.li, guang.hong, bijan.samali}@uts.edu.au

### **Abstract**

This paper presents a dynamic output feedback control approach to active control of civil engineering structures. The design is based on sliding mode control, pole placement of the closed-loop eigenvalues, and linear functional observers for regeneration of the equivalent control and the sliding function from output measurements. A five-storey model used for testing the proposed control scheme is described. Simulation results demonstrate its validity. Future work on structural control issues using the approach is outlined.

### 1 Introduction

The protection of civil structures, including their material contents and human occupants, from dynamic loadings such as hostile earthquakes, strong wind, extreme waves, heavy traffic, highway loadings is an issue of highest priority. Rapid developments in the field of feedback control and automation have successfully helped to mitigate these effects. A review of the recently developed control schemes and technologies can be found in [Spencer and Sain, 1997]. In active control of civil structures, the objective is to generate a proper control signal to drive the Active Mass Damper (AMD) actuator to react against the auxiliary mass and apply the inertial forces to reduce structural vibration responses in the desired manner. Various advanced control algorithms have been proposed for structural control, including the LQG or  $\hat{H}_2$  method [Spencer et al., 1994], neural network control [Wan et al., 1995], fuzzy control [Battaini et al. 1998], the  $H_{\infty}$  technique [Kose et al., 1996], polynomial control [Agrawal and Yang, 1997], singular value ( $\mu$ ) synthesis [Balas, 1998], sliding mode control [Wu et al., 1998], etc. A benchmark problem for active control of seismically-excited test models was presented in [Spencer et al., 1998].

Among efficient control techniques applied to wind and seismic response control, the variable structure system (VSS) with a sliding mode has demonstrated its strong robustness against disturbances and uncertainties [Yang et

al., 1997]. As known in structural control, it is impractical to install sensors on every degree of freedom (or floor) to measure the full-state vector. The design of output feedback controllers remains therefore an important issue in implementation of any control scheme. Furthermore, it is required to control only a few critical or dominant modes. For this, modal space sliding mode control has been recently proposed [Adhikari et al., 1998]. In this context, the problem of finding an explicit solution parameterizing the sliding surface such that the system closed-loop modes are placed as desirable is obviously of special interest [Edwards and Spurgeon, 1998]. For single input systems, an explicit form using Ackermann's formula for the sliding surface is derived in [Ackermann and Utkin, 1998].

This paper applies a dynamic output feedback sliding mode approach to the active control problem for a five-storey benchmark model, developed at UTS [Samali *et al.*, 2000] using pole placement and linear functional observers. The control design is rather straightforward and easy to implement. Simulation results using the proposed control scheme are presented in this paper. Work is in progress towards its real-time implementation.

### 2 The benchmark model

The experimental structure, shown in Figure 1, is a five-storey benchmark model, designed and manufactured at UTS, and approved by the *International Association for Structural Control (IASC)* for encouraging international collaborative research in the area of motion control of building structures [Samali *et al.*, 2000]. The model, of 1700 kg in total mass, is 3.6m tall with a footprint of 1.5m x 1.0m, and consists of two bays in one direction and a single bay in the other. The entire structure is tested on a 3x3 m, 6 tonne hydraulically-driven shake table. A simple AMD, shown in Figure 2, is placed on the 5<sup>th</sup> floor of the model, consisting of a single-ended hydraulic cylinder with steel masses attached to the clevis ends of the piston rod.

The actuator is controlled by an electrohydraulic directional proportional valve, sized accordingly for a maximum velocity of 1.65 m/s. The sensors used include

a load cell mounted at the actuator end cap to measure the control force, accelerometers positioned on the ground, each storey of the structure, and the AMD, and LVDTs placed between each floor and a fixed frame. The principal diagram for the experimental set-up is depicted in Figure 3.



Figure 1. Benchmark model



Figure 2. Active Mass Driver

## 3 Problem formulation

Consider an n degree-of-freedom building subject to either along-wind turbulence or one-dimensional earthquake. Assuming that the building is symmetric and there is no coupled lateral-torsional motion, the system dynamics can be represented by

$$M_{s}\ddot{x}(t) + C_{s}\dot{x}(t) + K_{s}x(t) = H_{u}u(t) + H_{w}w(t)$$
, (1)

where  $x(t) \in R^n$  is the vector of the displacement corresponding to each degree of freedom,  $u(t) \in R^m$  is the vector of control forces,  $w(t) \in R^q$  is the vector of wind or quake-induced forces applied to the structure,  $M_s, C_s, K_s$  are respectively  $(n \times n)$  mass, damping and stiffness matrices, and  $H_u \in R^{n \times m}$  and  $H_w \in R^{n \times q}$  are the matrices denoting the location of the controllers, and the wind or quake influence. In the state space form, (1) becomes

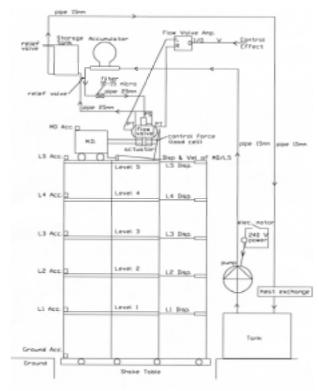


Figure 3. Experimental set-up diagram

$$\dot{X}(t) = AX(t) + BU(t) + Dw(t), \qquad (2)$$

where  $X(t) = [x(t), \dot{x}(t)]^T \in \mathbb{R}^{2n}$ , and matrices A, B, D are assumed to be known and given by

$$A = \begin{bmatrix} 0 & I_n \\ -M_s^{-1}K_s - M_s^{-1}C \end{bmatrix}, B = \begin{bmatrix} 0 \\ M_s^{-1}H_u \end{bmatrix}, D = \begin{bmatrix} 0 \\ M_s^{-1}H_w \end{bmatrix}.$$

The *r*-dimensional output vector, obtained from measurements, is expressed as

$$y(t) = CX(t), (3)$$

where  $C \in \mathbb{R}^{r \times 2n}$  [Yang *et al.*, 1997]. The objective is to design a sliding mode controller such that the closed-loop system has desired modes and is insensitive to desired using only information from the system output.

## 4 Control design

The design of sliding mode control (SMC) for (2) includes the selection of a sliding function so that the sliding motion when restricted to the sliding surface is stable, and then the derivation of a control law to enforce sliding mode in the sliding surface. The sliding function

$$\sigma(X,t) = SX(t) = [\sigma_1(X,t) \ \sigma_2(X,t) \ \dots \ \sigma_m(X,t)]^T,$$
(4)

where  $S \in \mathbb{R}^{m \times 2n}$ , may be determined such that the sliding mode dynamics in the sliding surface

$$S = \{X \in \mathbb{R}^{2n} \mid \sigma = SX(t) = 0\},$$
 (5)

have (2n-m) desired eigenvalues  $\lambda_1, \lambda_2, ..., \lambda_{2n-m}$ . The

following control law is employed [Ha et al., 1999]:

$$u(t) = u_E(t) + u_R(t)$$
, (6)

where  $u_E(t)$  is the equivalent control that may be obtained from a conventional method of the linear system theory applied to the nominal system, and  $u_R(t)$  is the robust control, which is switching in nature, developed to guarantee the reaching condition

$$\sigma^T \dot{\sigma} < 0. \tag{7}$$

## 4.1 Pole placement-based SMC

In the following a sliding mode controller will be designed with the sliding matrix  $S \in R^{m \times 2n}$  chosen by eigenvalue placement. If the sliding mode is enforced in the surface (5) then the system dynamic properties are determined by (2n-m) desired *sliding* eigenvalues  $\lambda_1, \lambda_2, ..., \lambda_{2n-m}$ . As noted in [Ackermann and Utkin, 1998], the design of the sliding surface does not generally imply assigning the rest m eigenvalues, which can take any arbitrary value. In this paper we make use of this design freedom to place them at  $\lambda = \lambda_*$ , where  $\lambda_*$  is some stable eigenvalue called the *sliding margin*. Using any pole placement algorithm, a state feedback control law of the form

$$u_F(t) = FX(t) \tag{8}$$

can be found for the equivalent control to assign the desired eigenstructure  $\{\lambda_1, \lambda_2, ..., \lambda_{2n-m}, \lambda_*, ..., \lambda_*\}$  such that:

$$\det(\lambda I - A^*) = (\lambda - \lambda_1)(\lambda - \lambda_2)...(\lambda - \lambda_{2n-m})(\lambda - \lambda_*)^m, (9)$$

where  $F \in \mathbb{R}^{m \times 2n}$  is the state feedback control matrix, and  $A^* = A + BF$  is the closed-loop matrix. Matrix S in (5) will be chosen such that

$$S(A + BF) = \lambda_* S , \qquad (10)$$

or  $\Omega S^T = 0$ , where  $\Omega = (\lambda_* I_{2n} - A^*)^T$ . A solution can be found in the form

$$S = N_{\Omega}^{T}, \qquad (11)$$

where  $N_{\Omega}$  is any basis of the null space of  $\Omega$ ,  $\ker(\Omega) = \ker\{(\lambda_* I_{2n} - A^*)^T\}$ .

In order to induce a sliding mode with the control law (6) where the equivalent control (8) is obtained from placing the closed-loop desired eigenstructure  $\{\lambda_1, \lambda_2, ..., \lambda_{2n-m}, \lambda_*, ..., \lambda_*\}$  for the nominal system, the robust control is given by

$$u_R(t) = -\eta \frac{B^T S^T \sigma}{\left\| B^T S^T \sigma \right\|} - \frac{B^T S^T S D}{\left\| S B \right\|^2} w, \qquad (12)$$

where  $\eta > 0$  is a coefficient denoting the convergence rate.

**Remark 1**: The first term of the robust control (12) is the

switching component that helps the system cope with actuator uncertainties  $\delta_{u}(t)$  [Alt *et al.*, 2000]:

$$u_{i,actual}(t) = [1 + \delta_{u_i}(t)]u_i(t),$$
 (13)

where  $\left|\delta_{u_i}(t)\right| < \eta$ , i = 1, 2, ..., m.

**Remark 2**: The second term of the robust control (12) offers a feedforward compensation for the influence of wind or quake-induced forces applied to the structure, which can be measured or estimated.

**Remark 3**: It can be shown that under the control laws (6), (8) and (12) the state vector X(t) asymptotically converges to zero and  $\lambda_1, \lambda_2, ..., \lambda_{2n-m}$  are the sliding eigenvalues.

#### 4.2 Linear functional observers

Examining the above control laws reveals that it could be implemented by using estimates of the equivalent control and the sliding function. Without loss of generality, let us assume that matrix C has full row rank, i.e. rank(C) = r, and takes the following canonical form

$$C = \begin{bmatrix} I_r & 0 \end{bmatrix}, \tag{14}$$

using, for example, the following transformation matrix

$$T = [C^{T}(CC^{T})^{-1} N_{C}], (15)$$

where  $N_C \in R^{2n \times (2n-r)}$  is any basis of  $\ker(C)$ . Let the feedback control matrix  $F \in R^{m \times 2n}$  be partitioned as

$$F = KL + WC, (16)$$

where  $K \in \mathbb{R}^{m \times p}$ ,  $L \in \mathbb{R}^{p \times 2n}$ , and  $W \in \mathbb{R}^{m \times r}$  are real matrices to be determined. Consider now a dynamical output feedback described by

$$FX(t) = (KL + WC)X(t) = Kz(t) + Wy(t),$$
  

$$\dot{z}(t) = Ez(t) + LBu(t) + Gy(t),$$
(17)

where  $z(t) = LX(t) \in \mathbb{R}^p$  is the state vector of the observer of order p,  $G \in \mathbb{R}^{p \times r}$  is a real constant matrix to be determined, and  $E \in \mathbb{R}^{p \times p}$  is a stable matrix to be selected according to the observer desired dynamics. Equation (17) can act as a dynamic output feedback controller to reconstruct  $X_f(t) = FX(t)$  provided that matrix E is chosen to be stable, and matrices G and E fulfil the following constraints [Trinh and Ha, 2000]:

$$\begin{cases}
GC - LA + EL = 0, \\
LD = 0, \\
F = KL + WC.
\end{cases}$$
(18)

As matrix E can be selected according to the desired dynamics of the observer, there are four unknown matrices (G, L, K and W) in system (18) to be solved for. Using (14) and the partition

$$F = [f_1 \quad f_2 \quad \dots \quad f_r \mid f_{r+1} \quad f_{r+2} \quad \dots \quad f_{2n}], (19)$$

where  $f_j = \begin{bmatrix} f_{1,j} & f_{2,j} & \dots & f_{m,j} \end{bmatrix}^T \in \mathbb{R}^m$   $(j = 1,2,\dots,2n)$  is the *j*-th column of F, it is shown that (18) is equivalent to [Trinh and Ha, 2000]:

$$\Pi l = \varphi , \qquad (20)$$

where  $l = \begin{bmatrix} l_1^T & l_2^T & \dots & l_{2n}^T \end{bmatrix}^T \in R^{2pn}$  with  $l_j \in R^p$  being the *j*-th column of *L*, and

$$\Pi = \begin{bmatrix} \Phi \\ \Psi \\ \Theta \end{bmatrix}, \ \varphi = \begin{bmatrix} f \\ 0 \\ 0 \end{bmatrix}, \tag{21}$$

where  $\Phi = [0_{\{m(2n-r)\}\times(pr)} \quad diag(K)_{\{m(2n-r)\}\times\{p(2n-r)\}}]$ , matrix  $\Psi \in R^{p(2n-r)\times 2pn}$  is given by

$$\Psi = \begin{bmatrix} a_{1,r+1}I_p & a_{2,r+1}I_p & a_{2,r+1}I_p & a_{2,n+1}I_p \\ a_{1,r+2}I_p & a_{2,r+2}I_p & & & & & & & \\ & & & & & & & & & & \\ a_{1,2n-1}I_p & a_{2,2n-1}I_p & & & & & & & & \\ a_{1,2n-1}I_p & a_{2,2n-1}I_p & & & & & & & & \\ a_{1,2n-1}I_p & a_{2,2n-1}I_p & & & & & & & & \\ a_{1,2n}I_p & a_{2,2n}I_p & & & & & & & \\ a_{1,2n}I_p & a_{2,2n}I_p & & & & & & \\ a_{2n-1,2n}I_p & & & & & & \\ a_{2n-1,2n}I_p & & & & & \\ a_{2n-2,2n}I_p & & & & & \\ a_{2n-2,2n}I_p & & & & \\ a_{2n-2,2n}I_p & & & & \\ a_{2n-2,2n}I_p & & \\ a_{2n-2,2n$$

with  $a_{j,k}$  denoting the (j,k)-element of matrix A,

$$\Theta = \begin{bmatrix} d_{1,1}I_p & d_{2,1}I_p & \dots & d_{2n,1}I_p \\ d_{1,2}I_p & d_{2,2}I_p & \dots & d_{2n,2}I_p \\ & & & \dots & & \\ d_{1,q}I_p & d_{2,q}I_p & \dots & d_{2n,q}I_p \end{bmatrix} \in R^{(pq)\times(2pn)},$$

with  $d_{j,k}$  denoting the (j,k)-element of matrix D, and

$$f = \begin{bmatrix} f_{r+1}^T & f_{r+2}^T & \dots & f_{2n}^T \end{bmatrix}^T \in \mathbb{R}^{m(2n-r)}$$
.

System (17) can be constructed to generate asymptotically the feedback law  $X_f(t) = FX(t)$ , even when the influence of wind or quake-induced forces applied to the structure are unknown, provided that (i)  $p \ge \frac{m(2n-r)}{r-q}$ ,

and (ii) matrix  $\Pi$  defined in (21) has full row rank.

**Remark 4**: The development in this section can also be employed for reconstruction of a full state feedback designed by using optimal control.

### 4.3 Dynamic output feedback SMC

Let us now apply the results developed above for estimation of the equivalent control and the sliding function to implement sliding mode control for system (2). Assume that the following dynamic output feedbacks

$$\begin{cases} \dot{z}(t) = Ez(t) + LBu(t) + Gy(t) \\ \hat{u}_E(t) = Kz(t) + Wy(t), \end{cases}$$
 (22)

$$\begin{cases} \dot{z}_S(t) = E_S z_S(t) + L_S B u(t) + G_S y(t) \\ \hat{\sigma}(t) = K_S z_S(t) + W_S y(t), \end{cases}$$
 (23)

have been designed to obtain the estimates  $\hat{u}_E(t)$  and  $\hat{\sigma}(t)$  respectively of the equivalent control (8) and the sliding function (4). It can be shown that dynamical errors

associated with these estimates:

$$\hat{u}_E(t) = u_E(t) + Ke(t)$$
, (24)

$$\hat{\sigma}(t) = \sigma(t) + K_S e_S(t), \qquad (25)$$

where e(t) = z(t) - Lx(t) and  $e_S(t) = z_S(t) - L_Sx(t)$ , will be forced to zero asymptotically under the proposed dynamic output feedback sliding mode control determined by (6), (8), and (12) if the equivalent control (8) is replaced by estimate  $\hat{u}_E(t)$  given in (22) and the robust control by estimate  $\hat{u}_R(t)$  defined by

$$\hat{u}_{R}(t) = -[\eta + ||Ke(t)||] \frac{B^{T} S^{T} \hat{\sigma}}{||B^{T} S^{T} \hat{\sigma}||} - \frac{B^{T} S^{T} SD}{||SB||^{2}} w, (26)$$

where  $\hat{\sigma}$  is obtained from (25) with  $E_S = \lambda_* I_{p_S}$ .

**Remark 5**: As the eigenvalue  $\lambda_*$  should be chosen at least about (3-5) times the dominant roots of the sliding eigenvalues such that  $e_S(t) \to 0$  quickly enough with respect to the sliding mode dynamics.

## **Design Algorithm:**

Step 1. Choose eigenvalues  $\lambda_1, \lambda_2, ..., \lambda_{2n-m}$  according to the desired sliding dynamics and the sliding margin  $\lambda_*$ .

Step 2. Design a suitable state feedback controller F for the equivalent control (8).

Step 3. Select the sliding function (4) with matrix S satisfying condition (10) by using (11).

Step 4. Design the dynamic output feedback (22) for the equivalent control.

Step 5. Choose  $E_S = \lambda_* I_{p_S}$ , design the dynamic output feedback (23) for the sliding function.

Step 6. Formulate the robust control (26).

## 5 Simulation results

Consider the benchmark model and the AMD shown respectively in Figure 1 and Figure 2. The displacement vector is  $x(t) = [x_1, x_2, x_3, x_4, x_5, x_m]^T$ , where  $x_i$  (i = 1,...,5) are the ith level absolute displacement, and  $x_m$  is that of the AMD, the control input is the driving force of the hydraulic cylinder, and the output vector obtained from two sensors is  $y(t) = [x_5, x_m, \dot{x}_5, \dot{x}_m]^T$ . In our case, 2n=12, m=1, q=1, and r=4. The matrices A, B, D and C respectively in eqs. (2) and (3) can be obtained as

 $B=[0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 2.7\ -29.4]^T$ 

 $D=[0\ 0\ 0\ 0\ 0\ 0\ -1\ -1\ -1\ -1\ -1\ -1]^T$ , and

The model has six modes listed as below

Mode	Damping	Freq. (Hz)
-0.5 ± 176i	0.00285	28.01
$-0.5 \pm 137i$	0.00365	21.80
$-0.5 \pm 96.3i$	0.00521	15.33
$-0.507 \pm 55.4i$	0.00915	8.82
-1.16 ± 16.2i	0.0713	2.58
$-1.33 \pm 18.9i$	0.0702	3.01

The control design is as follows.

Step 1 and 2: As forcing the closed-loop modes having desired damping factors may result in unacceptably large gains, the LQR technique is used to obtain the feedback F=lqr(A,B,diag([1e5,1e4,1e3,1e2,1e2,1e1,1,1,1,1,1,1,1,1,1,1,1]),.4) =[ -23.2015 -30.7185 -38.0197 -34.6135 281.1515 -1.0327 -0.4727 -0.5999 -0.5769 -0.2838 4.5661 -1.5454], which is equivalent to place the desired poles at  $\{-0.61 \pm 175.54i, -1.14 \pm 137.24i, -2.14 \pm 96.36i, -3.47 \pm 55.46i, -2.48 \pm 17.27i, -9.02, -38.07\}$ . The sliding margin is chosen as  $\lambda_*$  =-38.07.

Step 3. The sliding function is calculated from (11) as  $S = \begin{bmatrix} -0.9873 & 0.0526 & 0.0046 & 0.0063 & -0.0821 & -0.0778 \\ 0.0000 & 0.0002 & 0.0007 & 0.0019 & 0.0213 & 0.0962 \end{bmatrix}$ .

Step 4: The observer order is chosen such that  $p \ge \frac{m(2n-r)}{r-q} = \frac{8}{3}$ , hence p=3. The dynamic output

feedback control (22) can be designed with a selection of  $E = 2\lambda_* I_3$  and  $K=[50\ 0\ 0]$  to obtain matrices L, G and W using the algorithm given in [Trinh and Ha, 2000].

Step 5: Choose  $E_S = \lambda_* I_{p_S}$ ,  $K_S = [5\ 0\ 5]$ , again matrices  $L_S$ ,  $G_S$ , and  $W_S$  can be obtained to reconstruct the dynamic output feedback for the sliding function (23) can be reconstructed.

Step 6: Assuming an uncertainty of 500 N in the actuator force, the coefficient  $\eta$  is chosen equal to 500. The robust control (26) is now ready to be reconstructed.

We firstly test robustness of the proposed control scheme by assuming a random actuator uncertainty of maximally 400 N,  $\delta_u(t) = 400 * rand(1)$ . The responses of the fifth storey displacement and velocity are shown in Figure 4. The maximal values of displacement and velocity are observed to be less than 0.5mm and 200mm/s, respectively. The uncertainty and control force responses are shown in Figure 5. The resulted control force has a maximal magnitude less than 1kN. Note that chattering can be reduced by using a boundary layer or replacing the signum in (26) by a smoother function [Ha, 1997].

We now test the developed controller under the northsouth component of the El-Centro quake signal recorded at Imperial Valley Irrigation District substation in El Centro, California, during the Imperial Valley, California earthquake of May 18<sup>th</sup>, 1940. The displacement and velocity responses of the top (fifth) storey are shown in Figure 6 with the peak values being respectively less than 30mm and 500mm/s.

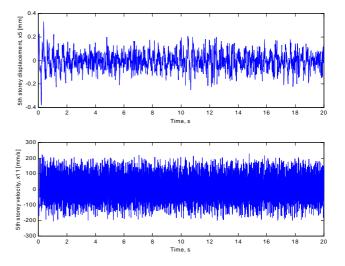


Figure 4. Displacement and velocity- storey 5.

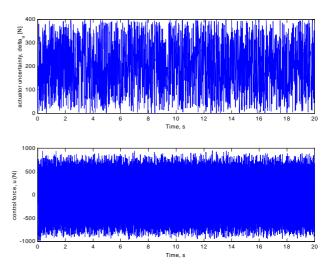


Figure 5. Actuator uncertainty and control force.

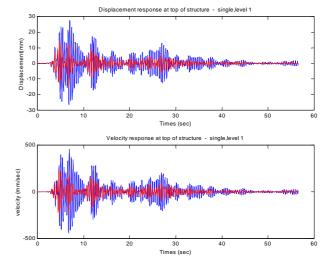


Figure 6. Top storey displacement and velocity: El-Ce quake

The acceleration and control force responses are shown in Figure 7, where a fuzzy technique has been used to cancel out the high frequency dynamics associated with the control output in the sliding mode.

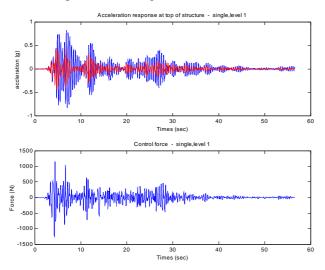


Figure 7. Acceleration and control force: El-Ce quake.

#### 6 Conclusion

We have presented a dynamic output feedback sliding mode control approach to the active control problem of civil engineering structures. The control signal consists of an equivalent control, designed based on pole-placement, and a robust control, comprising a switching component and a feedforward component. In comparison with the benchmark controller [Spencer et al., 1998], the proposed control scheme requires only information from the measured outputs using linear functional observers and can tolerate uncertainty of the actuator. Robustness and damping capability of the controller is verified through simulation results for a five-storey model developed at Work is in progress towards its real-time implementation on the model and further evaluation based on performance criteria adopted by the structural control community.

## Acknowledgements

The first author would like to acknowledge support from an IRG research grant and the UTS Centre for Built Infrastructure Research.

## References

- [Ackermann and Utkin, 1998] J. Ackermann and V.I. Utkin. Sliding mode control designed based on Ackermann's formula. *IEEE Trans. Automatic Control*, 43(2): 234-237, 1998.
- [Adhikari et al., 1998] J.C. Wu, J.N. Yang, and A.K. Agrawal. Applications of sliding mode control to benchmark problems. *Earthquake Engineering and Structural Dynamics*, 27(11): 1247-1265, 1998.
- [Agrawal and Yang, 1997] A.K. Agrawal and J.N. Yang. Static output polynomial control for linear structures. *Journal of Engineering Mechanics*, 123(6): 639-643, 1997.

- [Alt *et al.*, 2000] T.R. Alt, F. Jabbari, and J.N. Yang. Control design for seismically excited buildings: sensor and actuator reliability. *Earthquake Engineering and Structural Dynamics*, 29: 241-257, 2000.
- [Balas, 1998] G.J. Balas. Synthesis of controllers for the active mass driver system in the presence of uncertainty. *Earthquake Engineering and Structural Dynamics*, 27(11): 1189-1202, 1998.
- [Battaini *et al.*, 1998] M. Battaini, F. Casciati, and L. Faravelli. Fuzzy control of structural vibration. An active mass driven by a fuzzy controller. *Earthquake Engineering and Structural Dynamics*, 27(11): 1267-1276, 1998.
- [Edwards and Spurgeon, 1998] C. Edwards and S.K. Spurgeon. Compensator based output feedback sliding mode controller design. *Int. J. Control*, 71(4): 601-614, 1998.
- [Ha, 1997] Q.P. Ha. Sliding performance enhancement with fuzzy tuning. *Electronics Letters*, 33(16): 1421-1423, 1997.
- [Ha et al., 1999] Q.P. Ha, D.C. Rye, and H.F. Durrant-Whyte. Robust sliding mode control with application. *Int. J. Control*, 72: 1087-1096, 1999.
- [Kose *et al.*, 1996] I.E. Kose, W.E.. Schmittendorf, F. Jabbari, and J.N. Yang.  $H_{\infty}$  active seismic response control using static output feedback. *Journal of Engineering Mechanics*, 122(7): 651-659, 1996.
- [Samali *et al.*, 2000] B. Samali, M. Al-Dawood, H. Nguyen, and G. Murphy. Design and performance verification of an active mass driver. *Proc. the 5<sup>th</sup> Int. Conf. on Motion and Vibration Control MOVIC'2000*, pages 837-842, Sydney, Australia, December 2000.
- [Spencer and Sain, 1997] B.F. Spencer and M.K. Sain. Controlling Buildings: A New Frontier in Feedback. *IEEE Control Systems Magazine*, 17(6): 19-35, 1997.
- [Spencer *et al.*, 1994] B.F. Spencer, J. Suhardjo and M.K. Sain. Frequency domain optimal control strategies for aseismic protection. *Journal of Engineering Mechanics*, 120(1): 135-158, 1994.
- [Spencer et al., 1998] B.F. Spencer, S.J. Dyke, and H.S. Deoskar. Benchmark problems in structural control: Part I- Active Mass Driver System. Earthquake Engineering and Structural Dynamics, 27(11): 1127-1139, 1998.
- [Trinh and Ha, 2000] H. Trinh and Q.P. Ha. Design of linear functional observers for linear systems with unknown inputs. *Int. J. Systems Science*, 31: 741-749, 2000.
- [Wan et al., 1995] Y.K. Wan, J. Ghassboussi, P. Venini, and K. Nikzad. Control of structures using neural networks. Smart Materials and Structures, 4: 49-57, 1995.
- [Wu et al., 1998] J.C. Wu, J.N. Yang, and A.K. Agrawal. Applications of sliding mode control to benchmark problems. Earthquake Engineering and Structural Dynamics, 27(11): 1247-1265, 1998.
- [Yang et al., 1997] J.N. Yang, J.C. Wu, A.K. Agrawal, and S.Y. Hsu. Sliding mode control with compensator for wind and seismic response control. *Earthquake Engineering and Structural Dynamics*, 26(11): 1137-1156, 1997.