Observer-Based Decentralized Approach to Robotic Formation Control

A.D. Nguyen[+], Q.P. Ha[+], S. Huang[+], H. Trinh[++]

[+] ARC Centre of Excellence in Autonomous Systems
Faculty of Engineering,
University of Technology, Sydney,
PO Box 123 Broadway NSW 2007
E-mail: {adnguyen|quangha|sdhuang}@eng.uts.edu.au

[++] School of Engineering and Technology,
Deakin University,
Geelong, VIC 3217
E-mail: hmt@deakin.edu.au

Abstract

Control of a group of mobile robots in a formation requires not only environmental sensing but also communication among vehicles. Enlarging the size of the platoon of vehicles causes difficulties due to communications bandwidth limitations. Decentralized control may be an appropriate approach in those cases when the states of all vehicles cannot be obtained in a centralized manner. This paper presents a solution to the problem of decentralized implementation of a global state-feedback controller for N mobile robots in a formation. The proposed solution is based on the design of functional observers to estimate asymptotically the global state-feedback control signals by using the corresponding local output information and some exogenous global functions. The proposed technique is tested through simulation and experiments for the control of groups of Pinoneer-based non-holonomic mobile robots.

1 Introduction and background

The concept of multiple autonomous vehicles (land, air, or underwater) operating in formation is emerging as a key technology in mobile robotics that has been the focus of intense research effort over recent years. The formation control approach has several advantages over the traditional monolithic agent control, including overall system robustness, intelligence, enhanced performance (increased instrument resolution, reduced cost), and flexibility (reconfigurable capability, fault tolerance). Potential applications can be ranged from industrial coordination in agriculture, construction, and mining to diverse missions such as surveillance, wide-area search and rescue, environmental mapping, defense, and health care.

The pattern formation problem in multi-robot systems is defined as the coordination of a group of robots to get into and maintain a formation with a desired shape such as a wedge or a chain [Erkin *et al*., 2003]. Solutions for this problem are currently applied in search and rescue operations, landmine removal, remote terrain and space exploration, and also the control of satellites and unmanned aerial vehicles.

In forming and maintaining the multi-robot pattern there is generally a trade-off between precision and feasibility on one side, and between the necessity of global information and communication capacity on the other side. Those systems that require global information or broadcast communication may have a lack of scalability or high costs of the physical set-up but allow for more accuracy in forming a large range of robotic formations ([Sugihara and Suzuki, 1996], [Carpin and Parker, 2002]). On the contrary, systems using only local communication and sensor data, while limited in variety and precision of formations, tend to be more scalable, more robust, and easier to build ([Balch and Hybinette, 2000], [Desai, 2001]).

The robotic formation studies can be broadly classified into two groups. The first group includes cases where the coordination is done by a centralized unit that supervises the whole group and command the individual robots (see, e.g., [Egerstedt and Hu, 2001], [Koo and Shahruz, 2001], [Belta and Kumar, 2002]). Studies in the second group use distributed methods for achieving the coordination. This is the case when each control agent acts on the basis of local information and decisions (see, e.g., [Sugihara and Suzuki, 1996], [Yamaguchi *et al*., 2001], [Carpin and Parker, 2002]).

While there are different approaches to the robotic formation control problem, the common theme remains the global coordination of multi agents to accomplish intelligently and/or autonomously some task objectives. Fundamentally, approaches to multi-agent system control can be categorised into three broad groups: leader-following, behaviour-based, and virtual-structure [Beard *et al*., 2001].

In *leader-following*, one agent is designated as the leader, with the rest of the control agents designated as followers. The basic idea is that the followers track the position and orientation of the leader with some prescribed (possibly time-varying) offset. A survey on leader-follower techniques was given in [Cruz, 1978] for multi-level decision making systems. The application of these ideas to formation control for mobile robots was discussed in [Wang, 1991]. Several leader-following techniques have been proposed. Decentralised control laws are proposed in [Sheikholeslam and Desoer, 1992] for a special interconnection of a vehicle platoon in intelligent highways to maintain its cohesion. Spacecraft control using the leader following concept is reported in [De Queiroz *et al*., 2000] with adaptive control laws being proposed for keeping satellite formation in earth orbit. Feedback linearization techniques are used in [Desai *et al*., 2001] to derive tracking control laws for non-holonomic robots in a formation that is described as a directed graph. In [Carpin and Parker, 2002], the problem of leader following is addressed for the case of a multi-robot team with heterogeneous sensing capabilities.

The *behavioural* approach is based on the idea of prescribing for each agent several desired responses to possible excitations to the system, and making the control action for each agent a weighted average of the control for each response or behaviour. Behaviours in a multi-robot system may include forming, maintaining a formation, goal seeking, task execution, and collision/obstacle avoidance. The behavioural approach has been used to coordinate a formation of mobile robots to transport objects [Chen and Luh, 1994] or to control a group of robots in line and circle formations [Yun *et al.* , 1997]. A bebavior-based architecture is exploited in [Balch and Arkin, 1998] for multi-robot teams, where each local platform is controlled appropriately with respect to its neighbors by averaging several competing behaviours. To construct robotic formations, behavioural dynamics of heading direction and path velocity has been proposed recently [Monteiro *et al*., 2004], based on the concept of assigning dynamics to behaviours [Schoner *et al*., 1995].

In the *virtual structure* approach, the entire formation is treated globally as a single structure or so-called virtual structure. If the desired dynamics of the virtual structure can be translated into the desired motion of each agent then one can design local controllers to achieve global performance. The concept of virtual structure in the framework of cooperative robotics is introduced in [Lewis and Tan, 1997]. The virtual structure is applied to multiple spacecraft flying in [Beard *et al*., 2001], where, to achieve global coordination, knowledge of the virtual structure states is shared between each agent through dynamic coordination variables. Note that these variables are similar to the action reference notion introduced in [Kang *et al*., 2000] or the platoon-level functions given in [Stilwell, 2002].

Integration of all advantages of the three above-mentioned approaches has proven to be promising in coordinating multiple autonomous vehicles moving in formation. In [Beard, 2001], a control architecture for formation flying is proposed using formation and supervisor units in a centralized manner, and local controllers are designed to estimate the states of the local instantiations of these units. However, interconnections between formation and the local agents, interactions between the supervisor unit and behaviours are not introduced, and also the observer design mentioned in local control is not detailed. A framework for decentralized control of autonomous vehicles is proposed in [Stilwell and Bishop, 2000], using nonlinear observers to estimate the complete system state with minimal explicit communications between agents. The examples therein illustrate an autonomous platoon with a very simple model for the vehicle dynamics. A specific communication network topology is examined later in [Stilwell, 2002], where platoon-level functions representing global features that can be measured by an exogenous system. To implement these results in a realistic setting, a separate controller would be designed for each of a series of trajectories, and then the controllers would be gain scheduled as the vehicles move along the trajectories.

In moving toward a suitable architecture for multi-agent system control, this paper, motivated by [Stilwell, 2002] and [Ha and Trinh, 2004], is devoted to the decentralized implementation of a global state-feedback controller for a platoon of mobile robots in a formation under a decentralized information structure. The multi-agent system comprises generally N robots, each with a local control station. The control input for the ith station is calculated from the information contained in its local input and output signals only. Decentralised observers are also proposed here but unlike the approach by [Stilwell and Bishop, 2000], no explicit flow of information takes place among the control stations. The paper is organized as follows. After the introduction and background, section 2 presents the system description and formulates the problem. The main development of the proposed approach is detailed in section 3. The design procedure is illustrated in section 4 with simulation and experimental results included for a group of mobile robots. The platforms used for testing are the AmigoBOTs, shown in Figure 1.

2 Modelling

2.1 Model of a nonholonomic mobile robot

A mobile robot can be described by a common kinematic model as:

$$\dot{x} = v\cos\theta,\ \dot{y} = v\sin\theta,\ \dot{\theta} = \omega\,, \qquad (1)$$

where (x, y) is the center point on the wheel axis, $\theta \in R$ is the orientation and inputs v and ω are the translational and angular velocities respectively. The non-holonomic constraint of this system, implied in the model is

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0\,. \qquad (2)$$

Figure 1:Two Amigobots performing a line formation.

In general, the translational and angular velocities are limited by $|v| \le v_{\max}$ and $|\omega| \le \omega_{\max}$. This model is applicable for many indoor robots such as the Pioneers manufactured by ActivMedia Robotics (http://robots.activmedia.com). For outdoor vehicles wheel-to-ground friction should be taken into account in the modelling. By linearizing around a specific trajectory with $v = a = const$ and $\theta = b = const$ the corresponding velocities are $\dot{x} = a\cos b$, $\dot{y} = a\sin b$, and $\dot{\theta} = 0$. By selecting new variables

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{\theta} \end{bmatrix} = \begin{bmatrix} x-(a\cos b).t \\ y-(a\sin b).t \\ \theta - b \end{bmatrix}, \quad \begin{bmatrix} \bar{v} \\ \bar{\omega} \end{bmatrix} = \begin{bmatrix} v-a \\ \omega \end{bmatrix}, \tag{3}$$

one can obtain a linear time-invariant system for the robot as:

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{\bar{y}} \\ \dot{\bar{\theta}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -a\sin b \\ 0 & 0 & a\cos b \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{\theta} \end{bmatrix} + \begin{bmatrix} \cos b & 0 \\ \sin b & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{v} \\ \bar{\omega} \end{bmatrix}. \tag{4}$$

Note that this linearized model is valid when the motion of a robot in formation is near a specified trajectory [Stilwell, 2002].

2.2 Modelling a group of mobile robots in a formation

Consider a system composed of N mobile robots, uncoupled and modeled by (1). Each robot has a local controller that generates the local control signals based on local measured signals and signals broadcast exogenously. The states of the whole system can be described by:

$$X = [x_\Sigma \quad y_\Sigma \quad \theta_\Sigma]^T, \tag{5}$$

where $x_\Sigma = [x_1 x_2 \ldots x_N]^T$, $y_\Sigma = [y_1 y_2 \ldots y_N]^T$, $\theta_\Sigma = [\theta_1\theta_2 \ldots \theta_N]^T$, and where x_i, y_i, θ_i are positions and orientation of the *i*-th robot. The control input is $u = [v_\Sigma \;\; \omega_\Sigma]^T$, where $v_\Sigma = [v_1\, v_2 \ldots v_N]^T$ and $\omega_\Sigma = [\omega_1\, \omega_2 \ldots \omega_N]^T$, with v_i, ω_i being respectively the translational and angular velocities of the *i*-th robot, $i = 1,2,\ldots,N$.

Globally, there are features of the platoon that can be measured exogenously. These features such as the vehicle average position are referred to as action reference [Kang *et al*., 2000], dynamic coordination variables [Beard *et al*. 2001], or platoon-level functions [Stilwell, 2002]. In this paper, they are denoted $h(X)$, a function of the entire platoon state, assumed to be linear and differentiable, broadcast to all robots. In the control of a group of N mobile robots in a formation the state variables and platoon-level functions characterize the state of the overall system with respect to a global objective (i.e. getting into and maintaining a formation pattern). Thus, the model of the platoon can be written as:

$$\dot{S}(t) = AS(t) + Bu(t), \tag{6a}$$

$$y(t) = CS(t), \tag{6b}$$

where $S(t) = [\bar{X} \;\; h]^T \in R^n$ is the state vector, $\bar{X}$ is the global system state variables under consideration, $u(t) \in R^m$ and $y(t) \in R^r$ are the input and output vectors, respectively. Under the linearized conditions, matrices $A \in R^{n\times n}$, $B \in R^{n\times m}$ and $C \in R^{r\times n}$ are real constant.

Centralized control can be implemented if full information of S is made available to individual robots from a central unit. It becomes however very difficult when the size of the system is quite large. Following the approach proposed in [Stilwell, 2002], where the platoon-level functions representing integrated error signals are broadcast from an exogenous system, an alternative technique to the robotic formation control problem is proposed in this paper by using observer-based decentralized controllers.

3 Observer-Based Decentralised Control

Consider a linear time-invariant multivariable system described by (6). Without loss of generality, it is assumed that the triplet (A, B, C) is controllable and observable. Let N denotes the number of local control stations for N robots of the platoon. Let the elements of the input vector $u(t)$ and output vector $y(t)$ be arranged so that

$$u(t) = [u_1^T(t),\; u_2^T(t), \ldots,\; u_N^T(t)]^T, \tag{7a}$$

$$y(t) = [y_1^T(t),\; y_2^T(t), \ldots,\; y_N^T(t)]^T, \tag{7b}$$

where $u_i(t) \in R^{m_i}$ and $y_i(t) \in R^{r_i}$ $(i = 1,2,\ldots,N)$ are respectively the input and output vectors of the *i*-th agent (e.g., $u_i = [v_i\, \omega_i]^T$ in (5)). Accordingly, the system (6) can be rewritten as

$$\dot{S}(t) = AS(t) + \sum_{i=1}^{N} B_i u_i(t), \tag{8a}$$

$$y_i(t) = C_i S(t), \quad i = 1,2,\ldots,N \tag{8b}$$

where $B_i \in R^{n\times m_i}$ and $C_i \in R^{r_i\times n}$ are respectively sub-matrices of *B* and *C*, determined according to equations (7).

3.1 Assumptions

Let us first introduce some assumptions.

Assumption 1: The global system (A,B,C) is controllable and observable.

Assumption 2: There exist no decentralised fixed modes [Wang and Davison, 1973] associated with triplets (B_i, A, C_i), or if existing, they are assumed to be stable.

Assumption 3: Information available to the *i*th control station, $\Im_i(t)$, includes only the local output and control of the *i*th station:

$$\Im_i(t) = \{y_i(t),\ u_i(t)\},\quad i = 1,2,...,N\ . \tag{9}$$

Assumption 4: A satisfactory global state feedback control law has been found of the form

$$u(t) = FS(t), \tag{10}$$

where $F \in R^{m\times n}$, by using any standard state feedback control method to obtain the satisfaction of some system performance index.

Assumption 5: The conditions for obstacle/collision avoidance have been met.

3.2 Problem statement

Taking into account the constraint of the decentralised information structure (9), the objective here is to design decentralised controllers of the form

$$u_i(t) = f_i\{\Im_i(t), t\},\quad i = 1,2,...,N\ , \tag{11}$$

using only information available to local control stations, i.e. $\Im_i(t)$, such that the multi-robot system (8) is stable with satisfactory performance as prescribed in the global control law (10). To achieve the control objective the global control (10) will be constructed dynamically via decentralised linear functional observers that receive only $\Im_i(t)$ as their inputs.

Let the global controller (10) be partitioned as

$$u_i(t) = F_i S(t);\ i = 1,2,...,N\ , \tag{12}$$

where $F_i \in R^{m_i\times n}$. The decentralised controllers (11) are proposed to have the observer-based form:

$$u_i(t) = (K_i L_i + W_i C_i)S(t) = K_i z_i(t) + W_i y_i(t)\ , \tag{13a}$$

$$\dot z_i(t) = E_i z_i(t) + L_i B_i u_i(t) + G_i y_i(t), i = 1,2,...N\ , \tag{13b}$$

where $F_i = K_i L_i + W_i C_i$, $z_i = L_i S(t) \in R^{p_i}$ is the state vector of system (13); and real constant matrices $K_i \in R^{m_i\times p_i}$, $L_i \in R^{p_i\times n}$, $W_i \in R^{m_i\times r_i}$, $E_i \in R^{p_i\times p_i}$, and $G_i \in R^{p_i\times r_i}$ are to be determined.

3.3 Observer development

Let us assume, without loss of generality, that matrix C_i has full row rank, i.e. $rank(C_i) = r_i$, and takes the following canonical form

$$C_i = [I_{r_i}\quad 0]\ , \tag{14}$$

where I_{r_i} is an identity matrix of dimension r_i. Let the global control input matrix *B* be partitioned as

$$B = [B_i\quad B_{r_i}]\ , \tag{15}$$

where $B_{r_i} \in R^{n\times(m-m_i)}$. Accordingly, (6) can be expressed as

$$\dot S(t) = AS(t) + B_i u_i(t) + B_{r_i} u_{r_i}(t)\ , \tag{16}$$

$$y_i(t) = C_i S(t),\ \ i = 1,2,...,N\ , \tag{17}$$

where $u_{ri}(t)$ contains $(N-1)$ input vectors of the remaining $(N-1)$ control stations from other robots in the system.

Let an error vector $e_i(t)$ be defined as

$$e_i(t) = z_i(t) - L_i S(t);\ i = 1,2,...,N\ . \tag{18}$$

By some simple manipulations, the following error equation is obtained

$$\begin{aligned}\dot e_i(t) = \dot z_i(t) - L_i\dot S(t) &= E_i z_i(t) + L_i B_{r_i} u_{ri}(t) + G_i y_i(t)\\ &\quad - L_i AS(t) - L_i B_i u_i(t) - L_i B_{r_i} u_{ri}(t)\\ &= E_i e_i(t) + (G_i C_i - L_i A + E_i L_i)S(t) - L_i B_{r_i} u_{ri}(t)\end{aligned} \tag{19}$$

Therefore, (13b) can act as a decentralized linear functional observer for system (16-17), provided that matrix E_i is chosen to be asymptotically stable and matrices G_i and L_i fulfill the following constraints

$$\begin{cases} G_i C_i - L_i A + E_i L_i = 0 & (20)\\ L_i B_{r_i} = 0 & (21)\\ F_i = K_i L_i + W_i C_i . & (22)\end{cases}$$

Matrix E_i can be chosen according to the desired dynamics of the observer to be constructed. There are thus four unknown matrices (G_i, L_i, K_i and W_i) in equations (20)-(22) to be solved for. These matrices can be obtained exactly from a solution to the linear equation [Trinh and Ha, 2000]:

$$\begin{bmatrix}\Phi\\ \Psi\\ \Theta\end{bmatrix} l = \begin{bmatrix} f\\ 0\\ 0\end{bmatrix}, \tag{23}$$

where

$$\Phi = [0_{\{m_i(n-r_i)\}\times(p_i r_i)}\quad \Omega]\ , \tag{24a}$$

$$\Omega = diag\ \{K_i\} \in R^{m_i(n-r_i)\times p_i(n-r_i)}\ , \tag{24b}$$

$$l = \begin{bmatrix} l_1^T & l_2^T & \dots & l_n^T\end{bmatrix}^T \in R^{p_i n}\ , \tag{24c}$$

$$f = \begin{bmatrix} f_{r_i+1}^T & f_{r_i+2}^T & \dots & f_n^T\end{bmatrix}^T \in R^{m_i(n-r_i)}\ , \tag{24d}$$

in which

$f_j = \begin{bmatrix} f_{1,j}\ f_{2,j} \dots f_{m_i,j}\end{bmatrix}^T \in R^{m_i}$, $l_j = \begin{bmatrix} l_{1,j}\ l_{2,j} \dots l_{p_i,j}\end{bmatrix}^T \in R^{p_i}$ $(j = 1,2,..., n)$ are respectively the *j*-th column of matrices F_i and L_i, and where matrices $\Psi \in R^{p_i(n-r_i)\times p_i n}$ and

$\Theta \in R^{\{p_i(m-m_i)\}\times(p_i n)}$ are determined by

$$\Psi = \begin{bmatrix} a_{1,r_i+1}I_{p_i} & a_{2,r_i+1}I_{p_i} & \cdots & (a_{r_i+1,r_i+1}I_{p_i} - E_i) & \cdots & a_{n-1,r_i+1}I_{p_i} & a_{n,r_i+1}I_{p_1} \\ a_{1,r_i+2}I_{p_i} & a_{2,r_i+2}I_{p_i} & \cdots & . & \cdots & . & a_{n,r_i+2}I_{p_i} \\ . & . & \cdots & . & \cdots & . & . \\ a_{1,n-1}I_{p_i} & a_{2,n-1}I_{p_i} & \cdots & . & \cdots & (a_{n-1,n-1}I_{p_1} - E_i) & a_{n,n-1}I_{p_i} \\ a_{1,n}I_{p_i} & a_{2,n}I_{p_i} & \cdots & a_{r_i+1,n}I_{p_i} & \cdots & a_{n-1,n}I_{p_i} & (a_{n,n}I_{p_i} - E_i) \end{bmatrix} \tag{24e}$$

$$\Theta = \begin{bmatrix} b_{1,1}I_{p_i} & b_{2,1}I_{p_i} & \cdots & b_{n,1}I_{p_i} \\ b_{1,2}I_{p_i} & b_{2,2}I_{p_i} & \cdots & b_{n,2}I_{p_i} \\ . & . & \cdots & . \\ b_{1,(m-m_i)}I_{p_i} & b_{2,(m-m_i)}I_{p_i} & \cdots & b_{n,(m-m_i)}I_{p_i} \end{bmatrix} \in R^{\{p_i(m-m_i)\}\times(p_i n)}. \tag{24f}$$

Note that $\{m_i(n-r_i) + p_i(n-r_i) + p_i(m-m_i)\}$ linear simultaneous equations with $p_i n$ unknowns can be exactly solved if

$$\{m_i(n-r_i) + p_i(n-r_i) + p_i(m-m_i)\} \le p_i n, \tag{25}$$

the observer order p_i should therefore be chosen such that $p_i \ge \frac{m_i(n-r_i)}{r_i + m_i - m}$. If an exact solution to (23) is obtained then with E_i selected to be Hurwitz, error vectors $e_i(t)$ asymptotically approach zero. The local control laws (13a) will therefore reproduce asymptotically the global control (10).

Exact solutions to (13) may not always, however, be found, especially if low orders p_i of the observers (13b) are preferred. Alternatively, an approximate solution is procedure for solving matrices K_i and L_i [Ha and Trinh, 2004]. The procedure involves the formulation and solution of an optimisation problem, which will minimise the norm of the error between the two sides of equation (22) and (23). It is shown that the error norm of these two equations will determine the overall closed-loop stability of the system. The advantages of the approach include (i) the observers are completely decentralised in that each local control station uses locally available information only to generate the local control input signal, and hence, no information transfer among the local controllers required; and (ii) the order of the each local observer can be selected from a lowest value.

4 Design Illustration and Results

4.1 Modelling

For the illustration purpose let us consider a simple case of two mobile robots controlled in a 2-D formation parallel to the horizontal axis with a common absciss and a given average ordinate in a global Cartesian coordinate system. Here, the formation can be described by $x_1 = x_2$, $\frac{y_1 + y_2}{2} = 0$, and $\theta_1 = \theta_2 = 0$, where (x_1, y_1, θ_1) and (x_2, y_2, θ_2) are respectively the position and orientation of robot 1 and robot 2.

The global state vector of the form (6) is chosen as $S = [\bar\theta_1\ \bar\theta_2\ h_1\ h_2]^T$, where $\bar\theta_1 = \theta_1, \bar\theta_2 = \theta_2$, and the platoon level functions are $h_1 = x_1 - x_2 + \alpha(\theta_1 - \theta_2)$ and $h_2 = y_1 + y_2 + \beta(\theta_1 + \theta_2)$. Here functions h_1 and h_2 contain respectively the global information of the horizontal distance error between the robots, and the average value of the formation vertical position, and α, β represent the level of perturbation in distance measurements due to the robot orientation. Linearizing about the formation trajectory with $v = 2, \theta = 0$, one can obtain the system equation of the form (16):

$$\dot S = AS + B_1 u_1 + B_2 u_2, \tag{26}$$

where the local control inputs are

$u_1 = \begin{bmatrix} v_1 - 2 \\ \omega_1 \end{bmatrix}$, $u_2 = \begin{bmatrix} v_2 - 2 \\ \omega_2 \end{bmatrix}$,

(v_1, ω_1) and (v_2, ω_2) are the translational and angular velocities of robot 1 and robot 2 respectively, and

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & \alpha \\ 0 & \beta \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ -1 & -a \\ 0 & \beta \end{bmatrix}.$$

Here the decentralized information structure of the form (9) includes $y_1 = [\theta_1\ h_1\ h_2]^T$ and $y_2 = [\theta_2\ h_1\ h_2]^T$.

4.2 Observer Design

The global controller can be designed by using any available techniques in the control theory. For example, with $\alpha = 0.2, \beta = 0.5$, placing the closed-loop eigenvalues at {-1.5-1.5i; -1.5+1.5i; -0.8; -0.5} for the feedback control $u = FS$ yields

$$F = \begin{bmatrix} 0.0098 & -0.1411 & -0.25 & -0.0568 \\ -1.1826 & -0.1268 & 0 & -0.7958 \\ -0.0098 & 0.1411 & 0.25 & 0.0568 \\ -1.0851 & -1.5376 & 0 & -1.3638 \end{bmatrix}.$$

By applying the proposed method, observer–based decentralized observers of the form (13) can be obtained with:

Robot 1:

$$K_1 = \begin{bmatrix} 0.07055 & 0.07055 \\ 0.0634 & 0.0634 \end{bmatrix}, L_1 = \begin{bmatrix} 0 & 2 & 0 & -1 \\ 0 & 2 & 0 & -1 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} -0.25 & -0.339 & 0.0098 \\ 0 & -1.0494 & -1.1826 \end{bmatrix}, G_1 = \begin{bmatrix} 0 & 8 & 4 \\ 0 & 8 & 4 \end{bmatrix},$$

Robot 2:

$$K_2 = \begin{bmatrix} 0.0049 & 0.0049 \\ 0.054255 & 0.54255 \end{bmatrix}, L_2 = \begin{bmatrix} 0 & 2 & 0 & -1 \\ 0 & 2 & 0 & -1 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 0.25 & 0.0372 & 0.1411 \\ 0 & -3.534 & -1.5376 \end{bmatrix}, G_2 = \begin{bmatrix} 0 & 8 & 4 \\ 0 & 8 & 4 \end{bmatrix}.$$

4.3 Simulation results

With the specified formation, the desired positions and orientations are $x_{1d} = x_{2d}, y_{1d} + y_{2d} = 0, \theta_{1d} = \theta_{2d} = 0$. The following initial condition is chosen in our simulation:

$$\text{Robot 1: } \begin{pmatrix} x_{10} = -6 \\ y_{10} = 1 \\ \theta_{10} = 1(rad) \end{pmatrix}, \text{ Robot 2: } \begin{pmatrix} x_{20} = 15 \\ y_{20} = 9 \\ \theta_{20} = 2(rad) \end{pmatrix}.$$

Figures 2 and 3 depict the trajectories of robots with centralized and decentralized controllers respectively.

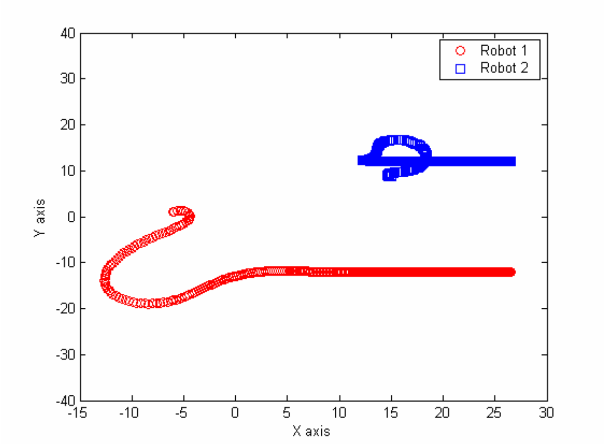

Figure 2: Robot trajectories with centralized control

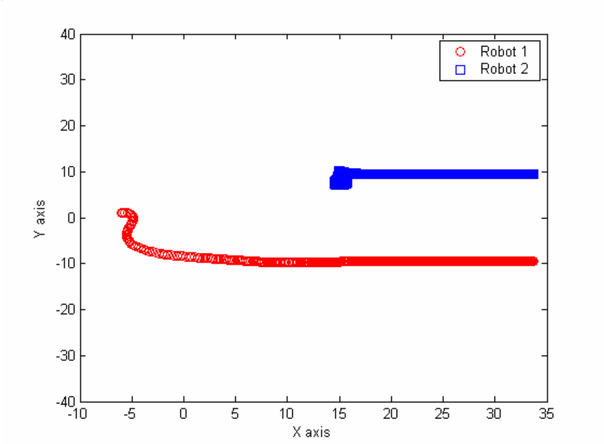

Figure 3: Robot trajectories with decentralized control

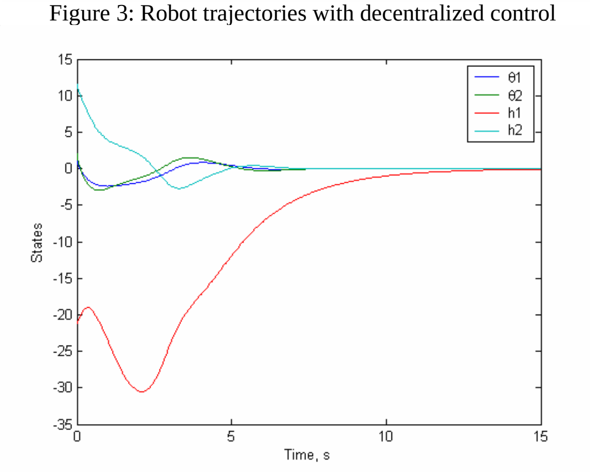

Figure 4: Global states of the system with centralized control

The global states $(\theta_1, \theta_2, h_1, h_2)$ of the systems are shown in Figs. 4 and 5, when controlled in both centralized and decentralized manner. Figures 6 and 7 present some snapshots over the time scale [0 15sec] of the multi-agent system with indices denoting the time points and dash lines representing the desired trajectories of the robots in the formation. Simulation has been conducted for three robots in wedge or parallel line/column formations. Fig. 8 shows the trace of three robots changing from a column to circle, circle to line, and from line to column formations.

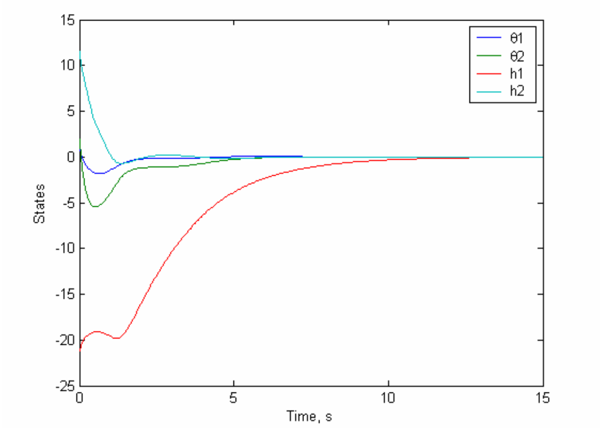

Figure 5: Global states of the system with decentralized control

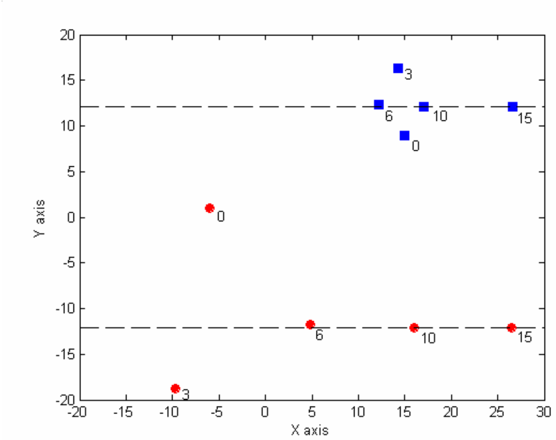

Figure 6: Snapshots over time with centralized control

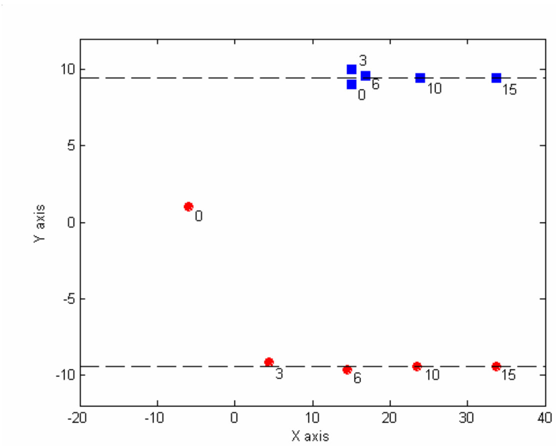

Figure 7: Snapshots over time with decentralized control

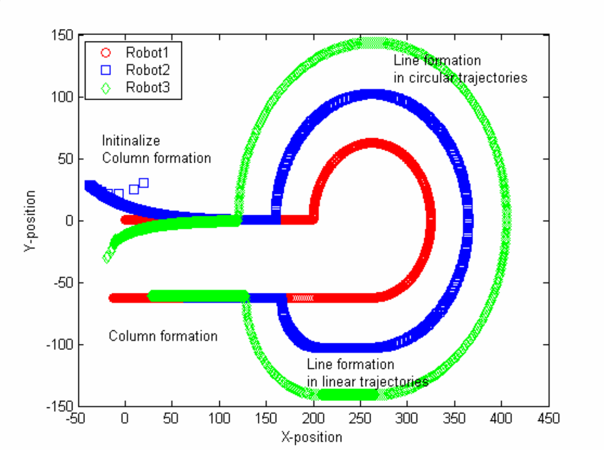

Figure 8: Three robots in various formations

It is clear from simulation results that the multi-robot system with the proposed decentralized controllers can form and maintain this simple desired formation. For controlling a more complicated formation, a piecewise linearization technique would be required. The simulation results are obtained in both MATLAB and the *Player/Stage*, a special software tool for Pioneers. Experimental work has been conducted to verify the proposed technique, as reported in the next section.

4.4 Experimental Results

Experimental platforms used for testing at the moment are the amigobots. Amigobot is a Pioneer-based mobile robot, constructed by ActivMedia Robotics. A photograph of the two mobile robots when maintaining a line formation is shown in Figure 1. The robot's architecture is widely adopted for coordination and control of multi robots. Range-finding is handled by six forward and two rear sonars mounted on the side of the robot. Shaft encoders track its local position in terms of (x, y, θ). Using differential drive and nearly holonomic design, the robot's mobility is acceptable over carpet edges and small sills. Information of sensing, motor and power monitoring and control is sent in packets over the wireless or tethered RS232 serial connection to PCs.

As described in the simulation, the objective of our experiments is to demonstrate the decentralized control algorithm for two Amigobots in entering and maintaining a simple line formation, from an arbitrary initial condition. Here, the control actions together with platoon level functions were computed from local information transferred from robots to the PC. The control inputs (translational and angular velocities) of each robot were computed in a decentralized manner. These signals were sent back through the wireless network to each robot. The control algorithm was programmed in C++ using ARIA classes.

The initial conditions of the two robots were expressed in a global coordinate frame as

Robot1 $(x_{10} = -400mm, y_{10} = -500mm, \theta_{10} = 90^0)$

and Robot2 $(x_{20} = -600mm, y_{20} = 600mm, \theta_{20} = 0^0)$.

In our experiments, the line formation for the two robots was two symmetrical straight trajectories parallel to the horizontal axis, as in the simulated results shown in Figure 3. The two robots should enter the formation parallel trajectories, located equidistantly to the horizontal axis.

Experimental results show that this simple robotic formation is formed and maintained successfully with two Amigobots, as can be viewed in the video clip supplied in the CD-Rom of the Conference. However, a small trajectory tracking errors may occur due to errors in position information, transferred from the encoders to the PC through the wireless communication. Tracking accuracy can be improved by using better sensors and by incorporating further navigation assistive algorithms for robot localization

Future research will aim to integrate the proposed approach into a suitable architecture that will allow for collision avoidance by using the artificial potential field with sliding mode control [Gulner and Utkin, 1995] and for formation fault-tolerance control by using optimization techniques such as the iterative Linear Matrix Inequality [Huang *et al*., 2002].

5 Conclusion

We have presented a solution to the problem of controlling a platoon of robots in a formation under a complete decentralized information structure. The proposed approach exploits decentralised linear functional observers to implement a suitable global feedback control law. Each local controller takes some global information of the formation from an exogenous unit and only local output measurements. The design technique is illustrated through the control of groups of Pinoneer-based mobile robots in different formations with simulation results provided. Experimental results reported illustrate the validity of the proposed technique for two Amigo robots in entering and maintaining a line formation from an arbitrary initial position.

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