Improving Early Numeracy Through a Pattern and Structure Mathematics Awareness Program (PASMAP)

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A school-based numeracy initiative, conducted in one NSW metropolitan primary school, trialled an innovative approach to improving mathematics for low achievers. The project involved 683 low-achieving students aged from 5 to 12 years, 27 teachers, and three researchers over a 9-month period. A Pattern and Structure Assessment (PASA) interview and a Pattern and Structure Mathematics Awareness Program (PASMAP) focused on improving students' visual memory, the ability to identify and apply patterns, and to seek structure in mathematical ideas and representations. There was a marked improvement in PASA scores particularly in the early grades and substantial improvements found in school-based and system-wide measures of mathematical achievement.

The early development of patterning and structural thinking in mathematics is critical to the abstraction and generalisation of mathematical ideas and relationships. Mathematical development depends to a large extent on the structure of students' thinking and how well this reflects the structure of mathematical concepts and relationships. For example, children need to recognise mathematical structure and how it is generalised in order to understand how the number system is organised by grouping in tens, and how equal groups form the basis of multiplication and division concepts.

Mathematics curriculum reform in Australia and internationally has draw attention to the importance of mathematical structure in early algebraic thinking and reasoning (Clements, 2004). For example, the NSW mathematics K-6 syllabus (Board of Studies, NSW, 2002) has recently incorporated a Patterns and Algebra strand from the first years of schooling. However, despite curriculum reform and a surge of new research, we have not yet formed a coherent picture of the early development of algebraic thinking and the potential impact that this may have on improving mathematics achievement generally. There are few studies on the development of young children's mathematical patterning, and on the development and effectiveness of early childhood programs promoting patterning skills. There is a real need to develop classroom-based studies that seek to identify teaching and learning influences that promote the development of structure and generalisation in children's mathematics learning. This may require researchers to pay more attention to the process of representation and abstraction and to the development of mathematical structure across various mathematical content domains. The issue of appropriate and effective professional development of teachers must also be integral to this research.

This paper reports one aspect of a large numeracy project — the effectiveness of a school-based implementation of a Pattern and Structure Mathematics Awareness Program (PASMAP) on student numeracy growth. The program was based on a research framework that described underlying features of pattern and structure as crucial to mathematical development (Mulligan, Prescott & Mitchelmore, 2004; Mulligan, Mitchelmore & Prescott, 2005).

Background to the Study

Recent studies have focused attention on the early bases of mathematical abstraction and generalisation with a number of major studies of early algebraic thinking (e.g. Blanton & Kaput, 2002; Schliemann, Carraher, Brizuela, Earnest, Goodrow, & Lara-Roth, 2003; Tabach & Hershkowitz, 2002; Warren, 2005). These studies indicate, that given appropriate opportunities, young children can develop early algebraic thinking and functional thinking from the early years of schooling. Studies of young children's mathematical reasoning and problem-solving investigations provide complementary evidence of the importance of early patterning skills and the development of structure in mathematical thinking (English, 2004; Fox, 2005). One such recent study (Papic & Mulligan, 2005) found that preschoolers participating in an intervention program promoting the development of patterning skills showed much greater improvement over a six-month period than the non-intervention children. The improvement was maintained was after a year of formal schooling.

Other researchers have extended previous studies on multiplicative reasoning to examine the use of pattern and structure in early numeracy (Mulligan et al., 2004; 2005). A descriptive study of 103 first graders, including 16 longitudinal case studies, found that children's perception and representation of mathematical structure generalised across a range of mathematical domains (counting, partitioning, unitising, patterning, measurement, space and graphs). They describe how children's mathematical concepts develop through five stages of structural development. Early school mathematics achievement was found related to children's underlying development and perception of mathematical pattern and structure. An implication of this research was that if low achievers had a poor awareness of pattern and structure, then their achievement could be improved if they were explicitly taught to recognise a variety of mathematical patterns and structures across mathematical content domains.

Method

A school-based numeracy project, supported by a NSW Department of Education and Training initiative was conducted with 683 students and 27 teachers over a one-year period beginning with teacher professional development in Term 4 and evaluation of the program in Term 4 of the following year. It was known from school-based, system-wide and standardised measures of mathematical achievement, that the majority of the students in the school had difficulties in acquiring basic mathematical concepts. Such measures included an analysis of data drawn from NSW Basic Skills Testing data and the Schedule for Early Number Assessment (NSW Department of Education & Training, 2002).

The development and implementation of the numeracy initiative focused on the following outcomes:

- A school-based numeracy program based on assessment and reporting of student achievement linked to teaching and learning strategies
- Effective individual teaching programs in numeracy through professional support and class release time for teachers for assessments, and classroom support in teaching and learning strategies
- Implementation of consistent interview-based assessments in numeracy across the school

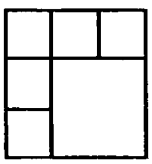
- Improved learning outcomes in numeracy for all students, particularly those considered most 'at risk'

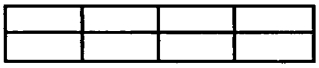
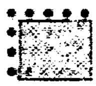
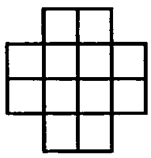
The implementation of the PASA assessment and PASMAP teaching program was based on the professional development model of Count Me In Too (CMIT) (NSW Department of Education and Training, 2002). This model enabled the integration of a structured and explicit framework of assessment tasks and teaching/learning strategies aimed primarily at developing underlying mathematical patterns and structures for low-achieving students. The project also supported and extended the implementation of CMIT in the school.

The Pattern and Structure Assessment (PASA)

A PASA interview-based assessment had been previously developed and trialled in several studies (Mulligan et al. 2004). The assessment focused on key mathematical concepts and processes such as visual memory, counting in multiples, number patterns, partitioning, unitising and multiplicative reasoning (see Table 1). Three PASA schedules were developed; one for each stage of learning — Early Stage 1, Stage 1 and Stage 2. The content areas of each of these schedules remained consistent although the level of difficulty of the tasks increased in terms of number size, complexity of problem-solving situation or type of representation. The number of tasks for Stages 1 and 2 increased from 20 in the Early Stage 1 PASA to 30 tasks in the schedules for Stages 1 and 2. These were reviewed, trialled and adjusted by stage-based groups of teachers before being administered to every student. Teachers were provided with release time to conduct PASA interviews and were provided with support and supervision from two researchers and the school's numeracy coordinator.

Table 1
Pattern and Structure Assessment (PASA) sample tasks

Number	Space	Measurement
Counting: multiples Count aloud in ones 1, 2, Count aloud by twos 2, 4 ... Count aloud by twos. Put a tick on the numbers (Use numeral tracks to 20, 30 etc.) Repeat process backwards. Use pattern of 3s.	*Unitising* Someone has started to draw some squares to cover this shape. Finish drawing the squares exactly the same size here 	*Clockface* Draw a clockface with everything you remember about a clockface drawn on it. Record various times e g 3 o'clock on it.
Empty Number Track Use wide strip of blank paper. Imagine this is a number track. If this end is zero and this end is 10 show me where 5 would be with the clip. Child allowed to adjust clip. .Record numbers 1-10 on the track. Does the child 'benchmark' 5 in the centre and use equal spaced units?	*Border Pattern* Complete the square border pattern of 20 tiles using two different colours. Use one colour for the corners. How many tiles do you need to complete the edges (4 on each side of the square)? Does the child count by ones each side? Identify corners? Recount corners?	*Make a Ruler* Imagine that you have to make a ruler for your friend so they can measure the length of small thing like your book. What would you draw on the ruler so they could measure? Draw as many things on the ruler as you need.

Reformulation of Fair Share There are 6 counters and 2 teddies. How many counters will each teddy get if we share fairly? Another teddy comes along. So there are now 3 teddies and 6 counters. How many counters will each teddy get?	*Unitising/partitioning* How many of these small triangles will fit exactly inside this shape with no gaps or overlaps (Show large triangle)	*Length Units* Place the sticks along the ribbon to show how long it is. How many sticks did you use? Does it matter where you start and finish? Does it matter if the sticks are different sizes?
Partitioning This is a picture of one block of chocolate.  Tell me how you would share all the chocolate fairly among 4 children so they get the same.	*Square Units* How many of these small squares (two) will fit exactly inside this shape with no gaps or overlaps? (Show large rectangle)	*Mass/Volume* Show three balls and ask the student to order them from lightest to heaviest. Ask for verification. Can a small ball be heavier than a large ball?. Explain.
Array: Multiplication I'm going to show you this card very quickly. How many dots are there? Open card, cover one row with hand and place in front of student. How many dots are under my hand? 	*2D/3D Visualisation/Volume* Imagine this shape folded up to make a box without a lid. How many cubes would fit in this box (without any spaces left)? 	*Length/ Fractions* Show me a half of this whole ribbon length. If we cut the ribbon how many pieces do we have? If we cut the ribbon into three pieces how many cuts do we have? Four pieces?

The Pattern and Structure Mathematics Awareness Program (PASMAP)

A framework for developing a structured numeracy program, informed by the PASA data, was then developed. The first researcher and the research assistant worked with the teachers to develop and implement structured learning experiences in Years K-5 that focused on key mathematical structures and patterns.

This included the classroom support of the researchers who provided teachers with practical materials and prepared activities. Figure 1 describes a sample of the activities used to develop pattern and structure. Many PASMAP activities developed students' visual memory as they observed, recalled and represented numerical and spatial structures in processes such as counting, partitioning, subitising, grouping and unitising. These activities were regularly repeated in varied form to encourage generalisation.

Procedures

Every teacher administered the initial PASA interview to all of their students and the results were used to allocate students to small groups for instruction. Results of the Term 1 PASA interview (February) were made explicit for each teacher with analysis of individual students' difficulties and strategies. Post-assessment (September) was conducted for those students who had been assessed in Term 1 and these were conducted by the project research assistants and the numeracy coordinator. The PASA data comprised PASA total scores, and students' strategies and drawn representations of solution processes. Data were summarised by the researchers for common patterns in response for individuals, and by individual teacher and stage of learning. This was made available for all teachers in the first school term. Additional diagnostic information was provided for teachers by the first researcher when requested. Comparative data was collected from individual students' SENAs and Year 3 and Year 5 NSW Basic Skills Testing results. Six teachers requested additional assistance with small group teaching and this was provided by the chief researcher and research assistant on a once-weekly basis.

Pattern Tasks: Reproduce patterns of simple grids and arrays of varying sizes, triangular or square number patterns

- Child reproduces model of simple pattern and explains their inaccurate image (Intuitive justification)
- Teacher shows correct and incorrect patterns and student determines correct model (Modelling)--
- How can we make your pattern the same as this one? Tell me why we are making it the same? Why is it the 'same' ? (Focus on 'sameness')
- Child's attention is drawn to shape, size, colour, equal-sized spaces (Focus on spatial or numerical structure)
- Screen each row, column or side of shape successively then child reproduces with counters or by drawing (Successive screening)
- Child justifies that the pattern is the same as the model (Justification)
- Child reproduces initial pattern from memory (Visual memory)
- Repeat task regularly (Repetition)

Border Patterns

- Use coloured tiles or unifix or multilink cubes to build linear pattern borders for example in two colours in different sized rectangular frames.
- Record the pattern using symbols of numbers. Complete a partially completed pattern.
- Describe the pattern using ordinals e g "Every third block is blue so I have x blue blocks in my pattern".
- Join the cubes at each corner to make a tall tower or horizontal strip of cubes. Can the student see that the pattern remains the same?
- Repeat the process and challenge students to record from memory.

Figure 1. PASMAP Teaching and Learning Activities

Several professional development meetings supported the planning and implementation of the PASA and PASMAP as well as assistance from the school's 'Learning Support' and 'Mentoring' teams. Additional numeracy support staff and practical resources were employed to enable effective implementation of the PASA and PASMAP. Following the PASA for all students and the analysis for each class PASMAP was implemented across the school in Terms 2 and 3. In effect, most teachers had only one full term of 10 weeks to implement PASMAP teaching strategies consistently to all students based on the diagnostic information provided by the PASA. It was envisaged that improvements in numeracy outcomes would take place on an ongoing basis and that teachers would develop and refine their individual program accordingly.

Discussion of Results

Most students made steady progress in developing patterning skills and structure in their mathematical thinking and representations. For example, the aim of one Year 1 activity was that students should learn that in a 2 x 3 rectangular grid of squares, the squares are of equal size, they touch each other along their sides, there is the same number in each row and in each column, and the total number can be counted. In one lesson, students who initially copied the grid using a scattering of open circles learned to use squares of a reasonable size showing some structure. In the PASA scheme of structural development, this meant that they moved from Level 1 (pre-structural) to Level 2 (emergent structure) (Mulligan et al., 2004). Further progress was made in the next lesson which repeated the exercise with different sizes and shapes.

The following data indicate marked improvement for students' PASA scores, particularly in the early grades. One aim of the PASA program was to target those students at the lower-achieving end of the range so further analysis of PASA scores was conducted for each class and individual student. The following figures summarise results for Early Stage 1, Stage 1 and Stage 2 using box and whisker plots. In these plots, the box covers the middle quartile and the whiskers extend to the maximum and minimum score.

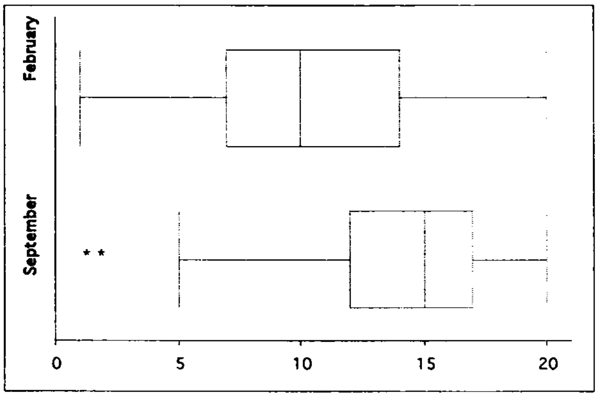


Figure 2. Early Stage 1 pre-assessment and post-assessment PASA scores

Figure 2 shows that the Early Stage 1 students made a marked improvement in PASA scores, with the median score increasing from 10 out of 20 in Term 1 to 15 out of 20 in Term 3. Most students with low scores showed improvements and the range of scores also narrowed so that most students scored between 12 and 18 out of 20.

Figure 3 shows marked improvement for all Stage 1 students with the median score increasing from 20 out of 30 to 24 out of 30. The range of scores narrowed with the lowest score in Term 3 at 11 out of 30 and most students scoring between 20-25.

Figure 4 shows marked improvement for all Stage 2 students with the median score increasing from 20 out of 30 in Term 1 to 24 out of 30 in Term 3.

Students with the lowest scores improved where the lowest achieving students in Term 3 scored 15 out of 30 rather than 10 out of 30. It was expected that Stage 2 students' results would produce a higher median than for Stage 1 students. Stage 2 students were largely drawn from Years 4 and 5 so they were expected to improve on the PASA more quickly as all the tasks were critical to achievement of Stage 1 and Stage 2 NSW Mathematics Syllabus outcomes. Substantial improvements were also found in school-based (class-based assessments of syllabus outcomes) and system-wide measures of mathematical achievement (NSW Basic Skills Test in Years 3 and 5 and the SENA 1 and 2 in Years K-2).

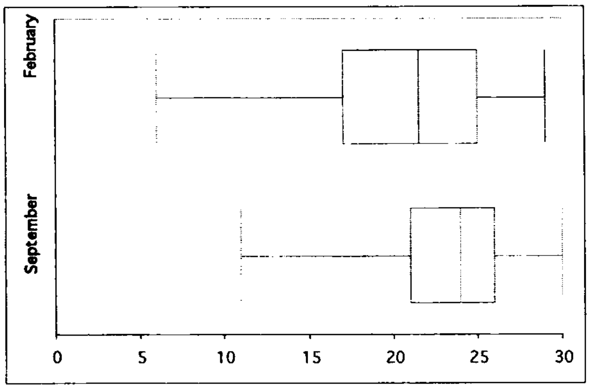


Figure 3. Stage 1 pre-assessment and post-assessment PASA scores

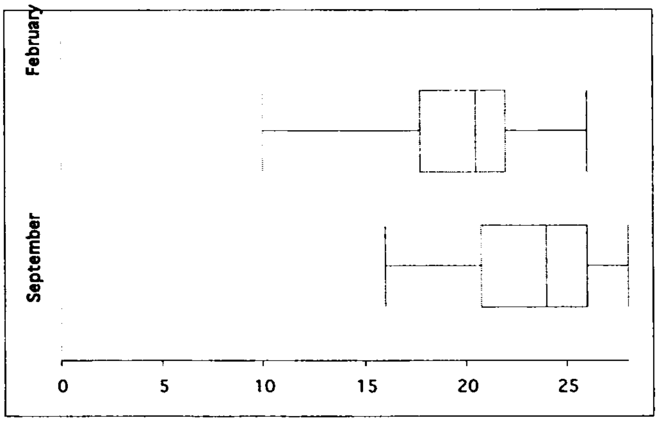


Figure 4. Stage 2 pre-assessment and post-assessment PASA scores

Conclusions and Limitations

This project was not a controlled study. It was limited to one school and there were too many threats to its internal validity to draw generalisations from the data. One of the advantages of the program was that teachers were initially involved in assessing their own students and were given support to analyse their difficulties and strategies. In the post-assessment there was insufficient support provided to release teachers to follow this through so they did not gain a first-hand picture of the substantial improvements that their own students had gained.

Nevertheless, the data suggest that explicit assessment and teaching of mathematical pattern and structure, delivered in a manner suitable for low-achieving students, has the potential to effectively improve students' acquisition of key mathematical concepts and processes development within a relatively short time frame. Data indicated that students generally lacked visualisation skills and had explicit difficulties with the transition from concrete to abstract mathematical thinking; many of these difficulties had not previously been detected by the teachers.

Further analysis of data provided by teachers and an independent evaluation (Koop,

2003) indicated that PASMAP had effectively addressed a range of student difficulties of which teachers were unaware. The study also highlights the critical importance of the type, level and consistency of teacher support in terms of academic assistance, additionally trained support staff and provision of practical resources. The analysis of observational and survey data indicated that the impact on the professional development of teachers was dependent on the level and continuity of teacher involvement and stage of learning. It appears that PASMAP had the most impact on those six teachers who participated in additional professional development activities where the researchers modeled learning experiences with small groups of students.

Encouraged by the study described in this paper, the authors have since trialled a revised form of PASMAP with increased attention to early algebraic reasoning in a one-year developmental design study with a Year 1 class. Further, a revision of PASMAP for Kindergarten students experiencing difficulties in mathematics learning is currently being developed. Wider dissemination studies are also planned.

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