Multilevel Optimization for Surface Mounted PM Machine Incorporating with FEM

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In this paper, Multilevel Genetic Algorithm (MLGA) is presented to solve the optimization of surface mounted permanent magnet synchronous machine (SPMSM), which has features of mixed continuous and discrete design variables, multi-modal and non-continuous objective functions, etc. Firstly, the multilevel optimization problem is described by using the problem matrix. The values in the problem matrix are deduced by correlation analysis. Secondly, the architecture and implementation of MLGA are carried out. Thirdly, the new algorithm is applied to a bi-level optimization of SPMSM to verify this multilevel optimization. The results compared with those of traditional genetic algorithm (GA) and discussions of the multilevel optimization are presented.

Index Terms—**Optimization, multilevel genetic algorithm, permanent magnet (PM) machine, finite element method (FEM).**

I. Introduction

According to the features and decision-making sequences, many real-world optimization problems in the engineering systems could be solved in multilevel procedures. The non-continuous design space, multi-modal objective functions, and mixed continuous and discrete variables are coexistent in one complex system.

Multilevel optimization is an effective method to solve this kind of complex optimization problem. It has been studied by some researchers. Bartheley [1] used the problem matrix method to describe the relationship between the objective functions and variables. Haftka [2] investigated two important problems in multilevel optimization: decomposition and co-ordination. In [3], the multilevel genetic algorithm (MLGA) was proposed and an actively controlled tower building subjected to earthquake excitations was considered to investigate the effectiveness of MLGA. Multilevel optimizations are difficult to solve due to the characteristics of nonlinearity, multi-modal functions and mixed continuous and discrete variables. Genetic algorithm (GA) can be used to solve multilevel optimization problem. However, the simple traditional GA can not handle the coupled relationship existing among the design variables, constraints and sub-problems.

Different optimization techniques have been developed for electric machine design to check iteratively the changes of the design variables, which move in the direction of improving the objective function. There are two main groups of optimization techniques: (1) Classical methods such as the direct search [4], the simplex method, and the Rosenbrock algorithm; (2) Stochastic methods such as the genetic algorithm and the simulated annealing technique. Some modern optimization techniques based on the fuzzy logic theory and artificial neural networks (ANN) [5] are also studied.

Numerical analysis, especially the finite element method (FEM), is a very powerful tool for performance analysis of electric machines, such as transient current, torque and velocity. The static FEM can also be used to determine the key parameters, such as magnetic flux linkage, electromotive force (EMF) and inductances, taking into account the details of complicated motor structures and the non-linear properties of magnetic materials. However, FEM is only an analysis tool and the design procedure is based on trial and error which is time-consuming and uncertain. The optimization of electromagnetic devices analyzed by FEM requires a high computing time. In additional, the parameterized structural modeling should be realized in the optimization procedure [6].

Permanent magnetic (PM) synchronous machines (PMSMs) are attractive choice for many applications because of their high efficiency and power density. In this paper, MLGA is presented for design optimization of SPMSM which has the features of mixed continuous and discrete variables, non-continuous space and nonlinear multi objective functions.

II. Formulation of Multilevel Optimization Problems

In multilevel optimization problems, the relationship between the design variables, constraints and objective functions can be described by the problem matrix, as shown in Fig. 1. The design variables may be assigned into different sub-vectors according to the relationships between design variable. The variables which have closed relationship should be allocated to the same sub-vector.

Fig. 1 represents the problem matrix which describes the relationship among the design variables, constraints and objective function. The symbols P_{ij} $(i=0,1,\ldots m,\ j=0,1,\ldots n)$, are the coefficients indicating the relative importance between the design variables and objective functions, as well as constraints in the correlation analysis [7]. The P value tests whether there is sufficient evidence that the correlation coefficient is not zero. The greater the P value is, the less relative importance of the design variable for the objective function is. In this paper, the samples of variables are determined by the design of experiment (DOE). Some commercial statistic software packages, such as SPSS and Minitab, can provide the module for relative importance analysis.

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Design variables	x_1	x_2	x_3	x_4	⋯	x_m
Objective function	P_{01}	P_{02}	P_{03}	P_{04}	⋯	P_{0m}
Constraint 1	P_{11}	P_{12}	P_{13}	P_{14}	⋯	P_{1m}
Constraint 2	P_{21}	P_{22}	P_{23}	P_{24}	⋯	P_{2m}
⋮			⋮			
Constraint n	P_{n1}	P_{n2}	P_{n3}	P_{n4}	⋯	P_{nm}

Fig. 1. Problem matrix

According to the *P* values in the problem matrix, the design variables may be arranged on diverse levels. For one objective function, the variables possessing similar *P* values will be managed on the same level.

III. Multilevel Genetic Algorithm

The traditional GA creates a vector (chromosome) encoded by all the design variables and then applies evolution operation to all the individuals described as chromosomes in one population. In MLGA the design optimization variables are classified and allocated to different levels according to the relative importance between the variables and objective functions, constraints, as well as the practical engineering weight and optimization sequence.

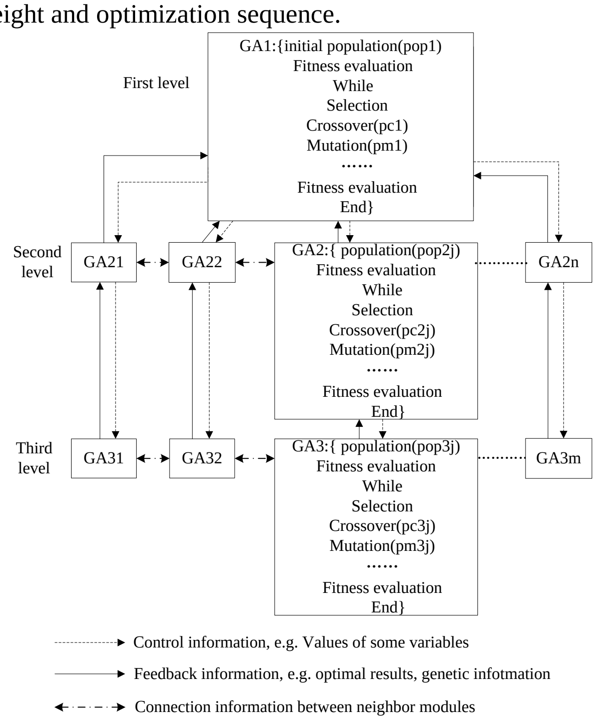

Fig. 2. Block diagram of MLGA.

The variables on different levels are encoded independently. Each level may have multiple populations and each of them can adopt different genetic operators and parameters. Furthermore, the relationship between sub-problems in multilevel problems can be handled by MLGA.

The architecture of MLGA is shown in Fig. 2. The upper level (GA_1) is the master GA module. The second (GA_{2i}) and third (GA_{3i}) consist of a number of modules. Each module corresponds to a sub-system. Owning to the interactions between the sub-systems on upper and lower levels, and as well as on the same level, a sub-system in the multilevel structures is not independent. The GA in one sub-system will be affected by other modules. The module in the upper level of the MLGA acts not only as a solver of the corresponding sub-problem, but also as a coordinator and controller of the modules on the lower level. This means that the lower level module GA_{ij} will be affected by the upper level module $GA_{i-1,j}$, and even by the adjacent modules $GA_{i,j-1}$ and $GA_{i,j+1}$ on the same level.

An independent GA can be described as follows.

$$GA=(\boldsymbol{PO}, \boldsymbol{PS}, \boldsymbol{IS}, \boldsymbol{FIT}, \boldsymbol{SO}, \boldsymbol{CO}, \boldsymbol{MO}) \tag{1}$$

where *PO*, *PS*, *IS*, and *FIT* represent the population, the population size, the encoding length and the fitness value, respectively; *SO*, *CO*, and *MO* are the genetic operations, i.e. selection, crossover and mutation.

The MLGA can be described as follows.

$$GA_{ij}=(\boldsymbol{PO}_{ij}, \boldsymbol{PS}_{ij}, \boldsymbol{IS}_{ij}, \boldsymbol{FIT}_{ij}, \boldsymbol{SO}_{ij}, \boldsymbol{CO}_{ij}, \boldsymbol{MO}_{ij}) \tag{2}$$

where GA_{ij} stands for applying the independent GA to the *i*th level and the *j*th module. In the view of the reaction between different levels and adjoint sub-modules on the same level, GA_{ij} can be described as follows.

$$\begin{aligned} GA_{ij}=(&\boldsymbol{PO}_{ij}(GA_{i,j-1}, GA_{i-1,j}, GA_{i,j+1}),\\ &\boldsymbol{PS}_{ij}(GA_{i,j-1}, GA_{i-1,j}, GA_{i,j+1}),\\ &\boldsymbol{IS}_{ij}(GA_{i,j-1}, GA_{i-1,j}, GA_{i,j+1}),\\ &\boldsymbol{FIT}_{ij}(GA_{i,j-1}, GA_{i-1,j}, GA_{i,j+1}),\\ &\boldsymbol{SO}_{ij}(GA_{i,j-1}, GA_{i-1,j}, GA_{i,j+1}),\\ &\boldsymbol{CO}_{ij}(GA_{i,j-1}, GA_{i-1,j}, GA_{i,j+1}),\\ &\boldsymbol{MO}_{ij}(GA_{i,j-1}, GA_{i-1,j}, GA_{i,j+1})\ ) \end{aligned} \tag{3}$$

The GA_{ij} can be affected by the upper level $GA_{i-1,j}$ or the same level modules, $GA_{i,j-1}$ and $GA_{i,j-1}$.

The implementation process of MLGA is as follows.

Step 1: Determine the objective functions, constraints and design variables.

Step 2: Analyze the relationship of design variables, objective functions and constraints by using correlation analysis, and construct the problem matrix.

Step 3: Determine the architecture of MLGA, including the number of levels and the number of modules in each level.

Step 4: Allocate the design variables, objective functions and constraints on different levels according to the problem matrix, and build up the relationships among different levels and different modules on each level. Each module corresponds to a genetic algorithm module.

Step 5: Implementation of MLGA starts from the top module of the MLGA, and then the modules in the lower level. The upper level module sends control messages and values of parameters to the lower level module. Feedback messages from the lower level are used as the evaluation function by the upper level.

Step 6: The total solving process ends when the termination criterion of the top level has been reached. Otherwise, Step 5 will be repeated.

The advantages of MLGA can be concluded as follows; The encoding of design variables on the lower level chromosome are modified with encoding of upper level chromosome. The parallel genetic operations performed in different modules within one level independently can enhance the diversity of the population. Every module is relatively independent to each other, which makes the genetic operators of selection, crossover, mutation, population size and number of evolution generations dynamiclly change in the implementation.

IV. Optimization Incorporating with FEM

In this paper, an SPMSM is optimized by using MLGA. In the optimization procedure, the static FEM is used to calculate the parameters with high precision.

Sometimes, MLGA may reduce the times of FEM calculation. For example, the thickness and width of permanent magnets are selected as the design variables on level 1 and the conductor number per slot and diameter of the conductors are assigned as the design variables on level 2. Other structural and material parameters are fixed. On level 1, the no-load EMF, d-axis and q-axis components of per turn inductances can be calculated when design variables are modified. On other levels, the thickness and width of permanent magnets are not taken as design variables, which are determined on level 1, the EMF, d-axis and q-axis components of inductances are proportional to the conductors per slot. In other words, FEM will not be conducted on level 2. If the total of populations and evolution generations of traditional GA are equal to those of MLGA, the computing cost of FEM in MLGA is less than that in traditional GA.

V. Optimization of SPMSM using MLGA Incorporating with FEM

An SPMSM under field oriented control (FOC), which is rated with output power of 950W, speed of 2000r/min and line-to-line voltage of 128V, is used to verify the MLGA to multilevel optimization. FOC controls the current space vector directly in the d-q reference frame of the rotor. One P-I controller drives the direct current component to zero and therefore the quadrature current produces useful torque, and maximizes the torque efficiency of the system. Another P-I controller operates on quadrature current and takes the requested torque as input.

The stator and rotor cores are not permitted to be modified due to manufacture limitation. The coil pitch, parallel branches and wires per conductor of the 3-phase windings are fixed. The magnet thickness and width, the diameter of conductor and the conductors per slot are chosen as the design variables. The optimization objective is to achieve the maximum efficiency with reasonable cost of conductors and magnets. The constraints are the fill factor and rated output power. The optimization model can be described as

$$\max\ f(\boldsymbol{X}) = K/(W_1 \cdot \frac{100-\eta}{100} + W_2 \cdot \frac{Cost(\mathrm{Cu})}{Max(\mathrm{Cu})} + W_3 \cdot \frac{Cost(\mathrm{PM})}{Max(\mathrm{PM})}) \quad (4)$$

s.t. $P_2 > 945\mathrm{W}$
$sf < 78\%$

where the design variable $\boldsymbol{X}$=[*hm bm Ns WindD*]; *hm* and *bm* are the magnet thickness and width, *Ns* and *windD* are the conductors per slot and the conductor diameter, which are all discrete variables. *Max*(Cu) and *Max*(PM) are possible maximum of the cost of stator windings and permanent magnets, respectively; *Cost*(Cu) and *Cost*(PM) represent the cost of stator windings and magnets, respectively; η is the efficiency of the SPMSM, K, W_1, W_2 and W_3 are weight factors defined by designer, P_2 is the output power, and *sf* is the fill factor.

The design variable $\boldsymbol{X}$ is a set of mixed continuous and discrete variables and $f(\boldsymbol{X})$ is a multi-modal objective function.

A. Determination of Multilevel Optimization Model

In this paper, the bi-level optimization model is chosen. The objective function and constraints (4) is shared in both levels. The fitness functions of both levels are the same, and the penalty function method is applied to deal with the constraints.

According to the theory of correlation analysis and DOE, the *P* values which represent the relative importance between design variables and object functions as well as constraints are analyzed by Minitab, a commercial statistic package.

The problem matrix is shown in Fig. 3.

Variables	*hm*	*bm*	*Ns*	*WindD*
max $f(\boldsymbol{X})$	0.270	0.666	0.001	0.000
$P_2 > 945$W	0.005	0.25	0.32	0.005
$Sf < 78\%$	1.000	1.000	0.000	0.000

Fig. 3. Problem matrix of MLGA for SPMSM

In Fig. 3, the *P* values of *Ns* and *WindD* are less than those of *hm* and *bm* with respect to objective function. *Ns* and *WindD* have important influences on efficiency and costs. Therefore, *hm* and *bm* are regarded as the variables of level 1 and *Ns* and *WindD* are assigned on level 2.

B. FEM for no-load EMF and L_{ad} and L_{aq}

On level 1, considering the nonlinear characteristics of the core, the static FEM is applied to calculate the no-load EMF per turn and the d- and q-axis components of inductances, i.e. L_{ad} and L_{aq}, per turn to acquire highly accurate parameters when the magnet thickness and width are changed. Before solving L_{ad} and L_{aq}, the nonlinear FEM should be conducted excited by permanent magnets only and the permeability of each finite element needs to be saved. When a linear FEM is applied to calculate L_{ad} and L_{aq}, the saved permeability will be assigned to corresponding elements. Fig. 4 illustrates the magnetic field distribution when L_{aq} is calculated. Fig. 5 shows the bi-level architecture of optimization for SPMSM.

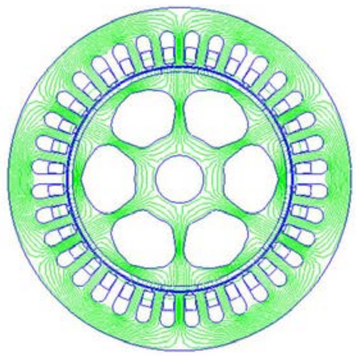

Fig. 4. Magnetic field distribution when L_{aq} is calculated.

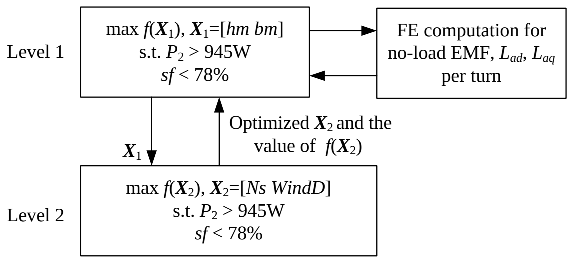

Fig. 5. Bi-level architecture of optimization for SPMSM.

C. Comparison between MLGA and traditional GA

Both the MLGA and traditional GA (single-level) are conducted for solving the optimization problem of SPMSM. The number of populations on levels 1 and 2 are 15 and 25, respectively. The number of evolution generations is 20 in each level. 40 populations and 40 evolution generations are defined in the single level GA. Table I lists the original design, the optimal results after MLGA and traditional GA.

TABLE I

COMPARISON BETWEEN MLGA AND TRADITIONAL GA

Variables and performances	Original design	Multilevel GA	Traditional GA
Thickness of PM, *hm* / cm	0.18	0.23	0.21
Width of PM, *bm* / cm	3.14	3.03	3.03
Conductors per slot, *Ns*	72	67	66
Diameter of conductor, *d* / mm	0.5	0.56	0.56
Back-EMF, *E0* / V	66.0	61.9	60.9
q-axis component of current, I_q / A	4.78	5.27	5.37
d-axis component of current, I_d / A	1.60	0.05	0.15
Efficiency, η (%)	83.7	86.4	86.1
Cost of winding / RMB	72.6	84.7	83.5
Cost of PM / RMB	41.3	50.9	45.5
Output power , P_2 / W	946	949.5	951
Fill factor, *sf* (%)	67	77.7	76.5

It can be seen that the optimized d-axis component of current is approached to zero in both MLGA and traditional GA and the optimal parameters of both GAs may be fit for using FOC. The efficiency optimized by the MLGA is higher than that optimized by the traditional GA. The higher the efficiency is, the higher cost of conductors and permanent magnets will be paid.

Fig. 6 illustrates the traces of fitness functions of MLGA and traditional GA. It can be seen that both MLGA and traditional single-level GA may achieve better results than the original design. The MLGA possesses better optimal fitness values than the single-level GA. It is suggested that MLGA can provide the better design solution because the number of populations in each level may be adjusted easily. In this case study, the GA operators have the same configures in both MLGA and single level GA. However, the designer may define appropriate GA parameters in different levels to search the satisfied optimum.

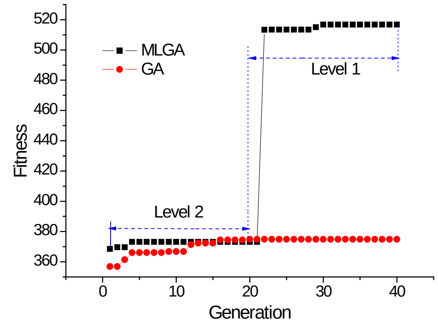

Fig. 6. Traces of fitness functions of MLGA and traditional GA.

VI. CONCLUSION

MLGA is presented and applied for solving mixed continuous and discrete and multilevel optimization problems of SPMSM design. It has a module-based architecture with each module corresponding to a sub-problem which makes it possible to handle the relationship between sub-problems in multilevel problems. The number of populations in each level may be adjusted to achieve satisfied optimums. The complex numerical calculation, such as FEM, may be conducted in partial levels, which may save the computing cost. Thus, the MLGA can be used to solve mixed continuous and discrete multilevel optimization problems effectively. Furthermore, the module-based architecture of the MLGA allows other conventional optimization techniques, e.g. PSO, to be included in some of the modules of the MLGA.

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