

**Physics of Society:** Econophysics and Sociophysics

# Interactive Macroeconomics

Stochastic Aggregate Dynamics with  
Heterogeneous and Interacting Agents

Corrado Di Guilmi • Simone Landini • Mauro Gallegati



## Physics of Society: Econophysics and Sociophysics

One of the major topics in macroeconomic theory is the coordination mechanism through which a large number of agents exchange goods in decentralized economies. The mainstream theory of efficient markets fails to provide an internal coherent framework and rules out, by construction, the interaction among heterogeneous agents. A rigorous micro-foundation for macro models can overcome this limitation and provide more reliable policy prescriptions. This book develops an innovative approach to the analysis of agent based models. Interaction between heterogeneous agents is analytically modelled by means of stochastic dynamic aggregation techniques, based on the master equation approach.

This book offers a systematic and integrated treatment of original concepts and tools, together with applications, for creating an alternative micro-foundation framework for a widespread adoption among the profession and by graduate students. In order to make the material accessible to graduate students, every non-standard mathematical tool or concept is introduced with a suitable level of detail. All the logical passages and calculations are explicit in order to provide a self-contained treatment that does not require prior knowledge of the technical literature.

**Corrado Di Guilmi** is Senior Lecturer at the University of Technology Sydney. His research interests include agent-based modelling, complex system theory, and post-Keynesian economics.

**Simone Landini** is Researcher at IRES Piemonte, Turin. His research interests include mathematical methods for socioeconomic and regional sciences, political and financial economics, agent-based modelling and complex systems theory.

**Mauro Gallegati** is Professor of Advanced Macroeconomics at the Università Politecnica delle Marche, Ancona. His research interests include heterogeneous interacting agents, agrowth and business fluctuations, financial fragility and complexity.

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## Interactive Macroeconomics

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The book is divided into four parts: the first presents the stochastic aggregation and macro-dynamics inference methods, based on the stochastic evolution of the microeconomic units; the second applies these inferential techniques on macroeconomic agent-based models; the third provides conclusions and stimulates further developments; the last part contains technical appendices.

This book offers a systematic and integrated treatment of original concepts and tools, together with applications, for the development of an alternative micro-foundation framework. In order to promote a widespread adoption among the profession and by graduate students, every non-standard mathematical tool or concept is introduced with a suitable level of detail. All the logical passages and calculations are explicit in order to provide a self-contained treatment that does not require prior knowledge of the technical literature.

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CDG.

*Alla mia famiglia, a mio cugino Stefano.*

*Agli amici, a Manfred.*

SL.

*Tra alti e bassi, la vita senza di te non mi appartiene.*

MG

# Contents

<i>Figures</i>	<i>xiii</i>
<i>Tables</i>	<i>xv</i>
<i>Preface</i>	<i>xvii</i>

## 1 Introduction

1.1 Why are We Here?	1
1.2 Aggregation and Interaction	7
1.3 The Road Ahead	12
1.4 Structure of the Book	21
1.4.1 Three possible reading paths	22

## Part I Methodological Notes and Tools

## 2 The State Space Notion

2.1 Introduction	29
2.2 The State Space Notion	31

## 3 The Master Equation

3.1 Introduction	46
3.2 The Master Equation: A General Introduction	49
3.2.1 The mechanics inside	49
3.2.2 On the meaning of the ME	53
3.3 The Markov Hypothesis	55

3.3.1 The simplest case	55
3.3.2 A generalization	57
3.3.3 Stationary solution	60
3.4 Moments Dynamics	62
3.4.1 Basics on moments	62
3.4.2 Exact dynamic estimators	64
3.4.3 Mean-field dynamic estimators	66
3.5 Concluding Remarks	67
<b>Part II Applications to HIA Based Models</b>	
<b>A Premise Before Applications</b>	71
<b>4 Financial Fragility and Macroeconomic Dynamics I: Heterogeneity and Interaction</b>	
4.1 Introduction	73
4.2 A Financial Fragility ABM	75
4.2.1 Goal of the model	76
4.2.2 Main assumptions	78
4.2.3 States of financial soundness	80
4.2.4 The microeconomic behaviour	81
4.2.5 The optimal programming rule	84
4.2.6 Macroeconomic effects and parametrization	88
4.3 Macroeconomic Inference of Stochastic Dynamics	97
4.3.1 The ME applied to the ABM	97
4.3.2 The ME transition rates from the ABM	101
4.3.3 The ME solution to the ABM	105
4.4 Results of Monte Carlo Simulations	108
4.4.1 The simulation procedure	108
4.4.2 Economic scenarios and inference	111
4.5 Concluding Remarks	131
<b>5 Financial Fragility and Macroeconomic Dynamics II: Learning</b>	
5.1 Introduction	133

5.2 A Financial Fragility ABM	136
5.2.1 Main assumptions	137
5.2.2 Goal of the model	139
5.2.3 The microeconomic behaviour	142
5.3 Macroeconomic Inference of Stochastic Dynamics	151
5.3.1 The ME applied to the ABM	153
5.3.2 The ME transition rates from the ABM	155
5.3.3 The ME solution to the ABM	161
5.4 Monte Carlo Scenarios and Simulation Results	161
5.4.1 The simulation procedure	161
5.4.2 Economic scenarios and inference	163
5.5 Concluding Remarks	190

### Part III Conclusions

#### 6 Conclusive Remarks

6.1 The Relevance of this Book	195
6.2 Current Work and Possible Future Developments	199
6.2.1 Thinking atoms	200
6.2.2 Towards a comprehensive representation of the economy	201

### Part IV Appendices and Complements

<i>Appendix A</i> Complements to Chapter 3	205
<i>Appendix B</i> Solving the ME to Solve the ABM	211
<i>Appendix C</i> Specifying Transition Rates	242
<i>References</i>	273
<i>Index</i>	283